Autumn
Scheme of learning

Year 10

#MathsEveryoneCan

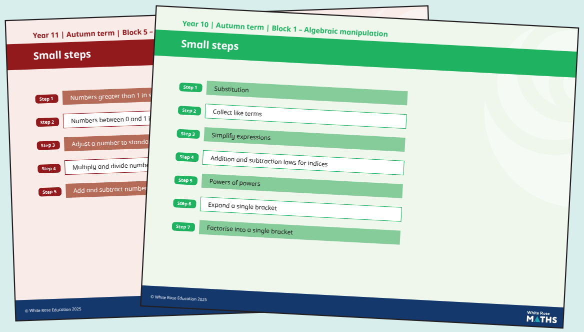
White Rose MATHS

The **White Rose Maths** schemes of learning

Why small steps?

We know that if too many concepts are covered at once, it can lead to cognitive overload, so we believe it is better to follow a small steps approach to the curriculum. As a result, each block of content in our schemes of learning is broken down into small manageable steps.

It is not the intention that each small step should last a lesson – some will be a short step withing a lesson; some will take longer than a lesson. We encourage teachers to spend the appropriate amount of time on each step for their group, and to teach some steps alongside each other if necessary.



Teaching for mastery

Our research-based schemes of learning are designed to support a mastery approach to teaching and learning and are consistent with the aims and objectives of the National Curriculum.

Curriculum structure and sequencing

Key concepts are introduced in a carefully considered logical sequence, allowing students to build a deep and connected understanding. This consistent and coherent approach supports students in making links within and between topics over time, laying secure foundations for future learning.

Evidence-informed design

Our curriculum is designed by a team of maths specialists, with an emphasis on how students learn, retain and retrieve knowledge. We incorporate strategies such as retrieval practice and spaced learning to support long-term memory and deepen understanding. The use of models and representations plays a key role in reducing cognitive load and making abstract concepts more accessible. Informed by the latest educational research, each scheme of learning also identifies common misconceptions and provides targeted approaches to address them, supporting effective, evidence-informed teaching in every classroom.

Fluency, reasoning and problem solving

Our schemes develop all three key areas of the National Curriculum, giving students the knowledge and skills they need to become confident mathematicians.

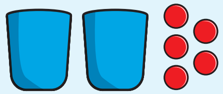
The **White Rose Maths** schemes of learning

Concrete – Pictorial – Abstract (CPA)

Research shows that all students, when introduced to a new concept, should have the opportunity to build competency by following the CPA approach. This features throughout our schemes of learning.

Concrete

Students should have the opportunity to work with physical objects/concrete resources, in order to bring the maths to life and to build understanding of what they are doing.



Pictorial

Alongside concrete resources, students should work with pictorial representations, making links to the concrete. Visualising a problem in this way can help students to reason and to solve problems.

Abstract

With the support of both the concrete and pictorial representations, students can develop their understanding of abstract methods.

$$2x + 5 = 13$$

Key Stage 3 and 4 symbols

The following symbols are used to indicate:

 concrete resources might be useful to help answer the question

 a bar model might be useful to help answer the question

 drawing a picture might help students to answer the question

 students talk about and compare their answers and reasoning

 a question that should really make students think. The question may be structured differently or require a different approach from others and/or tease out common misconceptions.

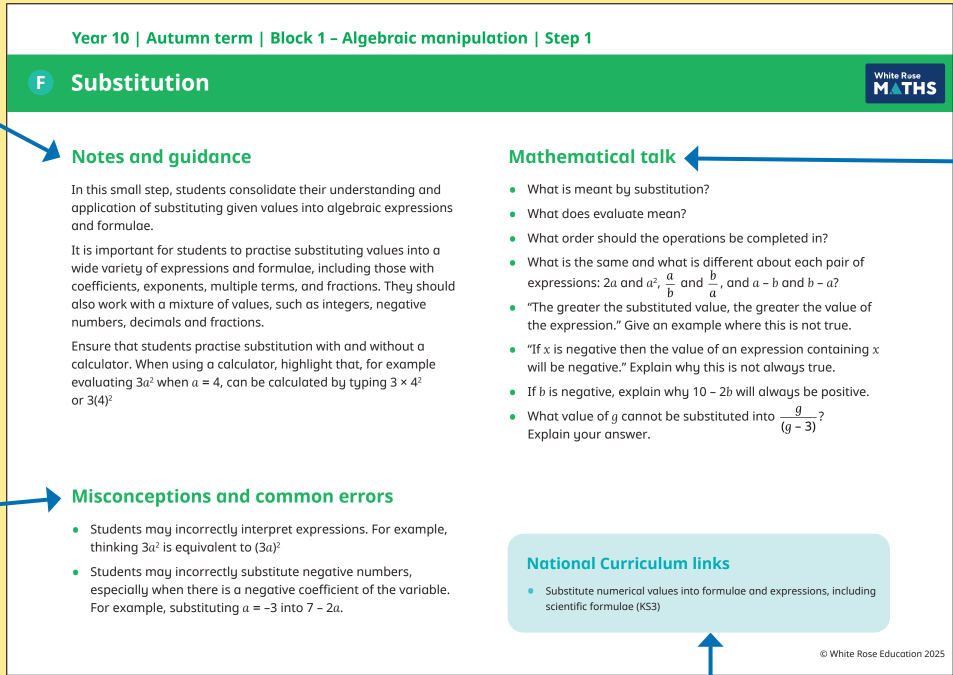
 the step has an explicit link to science, helping students to make cross-curricular connections.

Teacher guidance

Every block in our schemes of learning is broken down into manageable small steps, with comprehensive teacher guidance for each one. Here are the features included in each step.

Notes and guidance provide an overview of the content of the step, and ideas for teaching, along with advice on progression and where a topic fits within the curriculum.

Misconceptions and common errors are highlighted, as well as areas that may require additional support.



Year 10 | Autumn term | Block 1 – Algebraic manipulation | Step 1

Substitution

Notes and guidance

In this small step, students consolidate their understanding and application of substituting given values into algebraic expressions and formulae.

It is important for students to practise substituting values into a wide variety of expressions and formulae, including those with coefficients, exponents, multiple terms, and fractions. They should also work with a mixture of values, such as integers, negative numbers, decimals and fractions.

Ensure that students practise substitution with and without a calculator. When using a calculator, highlight that, for example evaluating $3a^2$ when $a = 4$, can be calculated by typing 3×4^2 or $3(4)^2$

Mathematical talk

- What is meant by substitution?
- What does evaluate mean?
- What order should the operations be completed in?
- What is the same and what is different about each pair of expressions: $2a$ and a^2, $\frac{a}{b}$ and $\frac{b}{a}$, and $a - b$ and $b - a$?
- "The greater the substituted value, the greater the value of the expression." Give an example where this is not true.
- "If x is negative then the value of an expression containing x will be negative." Explain why this is not always true.
- If b is negative, explain why $10 - 2b$ will always be positive.
- What value of g cannot be substituted into $\frac{g}{(g - 3)}$? Explain your answer.

Misconceptions and common errors

- Students may incorrectly interpret expressions. For example, thinking $3a^2$ is equivalent to $(3a)^2$
- Students may incorrectly substitute negative numbers, especially when there is a negative coefficient of the variable. For example, substituting $a = -3$ into $7 - 2a$.

National Curriculum links

- Substitute numerical values into formulae and expressions, including scientific formulae (KS3)

Mathematical talk provides key questions, discussion points and possible sentence stems that can be used to develop students' mathematical vocabulary and reasoning skills, digging deeper into the content.

National Curriculum links to indicate the objective(s) being addressed by the step.

Teacher guidance

Teaching approaches offer practical strategies for classroom use, including effective representations, modelled examples and key questions or activities designed to promote reasoning and problem solving.

Year 10 | Autumn term | Block 1 – Algebraic manipulation | Step 1

F Substitution

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Teaching approaches

- Display cards showing expressions and values.

$\frac{b}{c} - a$	$b - \frac{a}{c}$	$\frac{b - a}{c}$	$a = 0.5$	$b = -2$	$c = 4$

Ask students questions to support them with the substitution.

- Which expressions require a division before the subtraction?
- Which expressions will be positive? Which will be negative?

Repeat with different expressions and values.

- Show students a formula.

A rugby union team scores T points in a match.

$$T = 7c + 5u + 3p$$

c = number of converted tries scored
u = number of unconverted tries scored
p = number of penalty kicks or drop goals scored

Ask students questions about the formula.

- What is the value of T if $c = 2$, $u = 1$, $p = 5$?
- If $T = 15$, what could c, u and p be?

Key vocabulary

expression	collection of terms involving variables (letters) and numbers
variable	symbol, usually a letter, that can represent any value in mathematical expressions, identities and formulae
formula	mathematical rule or equation that shows the relationship between different variables or quantities
substitute	replace letters with numerical values

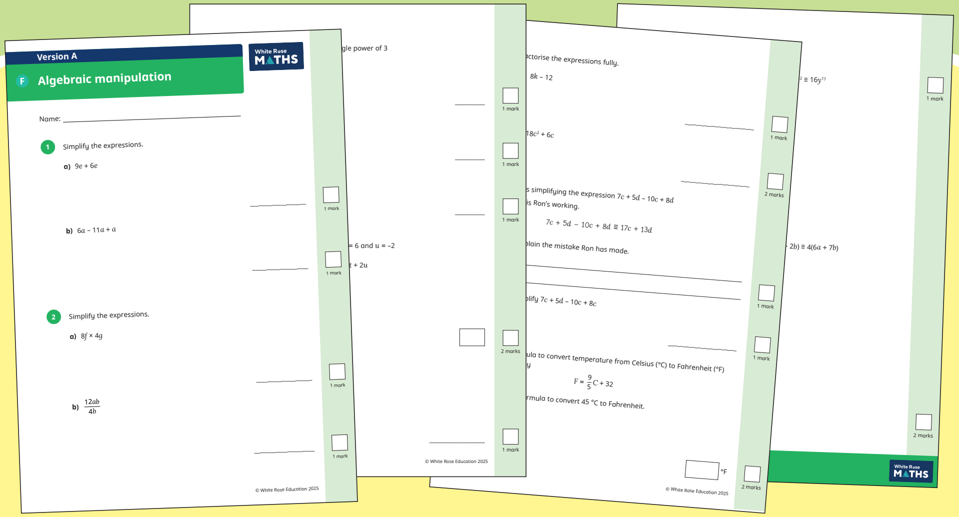
Links and next steps

- Students will substitute into a variety of scientific formulae, such as Newton's Second Law.
- Support Curriculum – Year 8 Spring Block 5 – Step 4 – Substitution
- Main Curriculum – Year 9 Autumn Block 4 – Step 6 – Substitute into formulae and equations
- Being fluent in substitution is an essential technique and is a part of the process of solving a pair of linear simultaneous equations.

Key vocabulary emphasises the importance of mathematical language and offers clear, age-appropriate definitions to support understanding

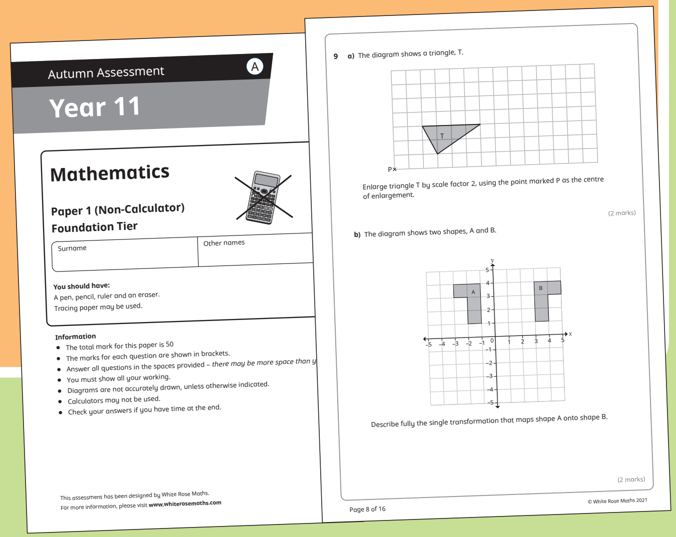
Links and next steps highlight connections to science (where appropriate) as well as alignment with the Support curriculum and shows how this step builds towards future learning. It may also include a challenge to deepen understanding, while remaining within the scope of the small step.

Free supporting materials



End-of-block assessments are provided for teachers to see how well students are progressing with the material in the curriculum. These have a total of 20 marks, assessing students' understanding of all of the steps within a block. These can be used flexibly – in the classroom, as homework, with/without a calculator, immediately after a block or later in the year – to suit teachers' and students' needs. Answers are provided.

End-of-term assessments are also provided for teachers to assess how well material is being learnt and retained in the medium and long term. There will be a calculator and non-calculator paper provided for the end of each term for each Years 7, 8 and 9. All papers will have a total of 40 marks available. We suggest 45 minutes for a paper, so that they can be done within a typical lesson. Mark schemes are provided.



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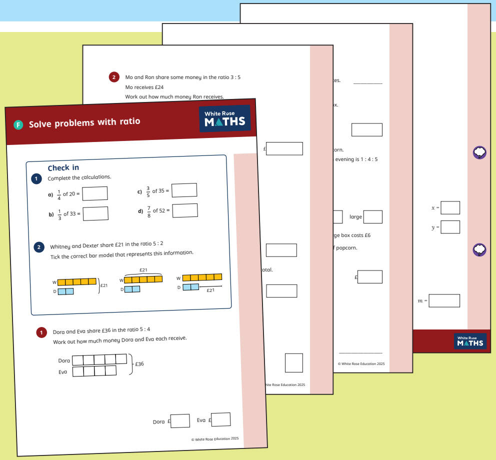
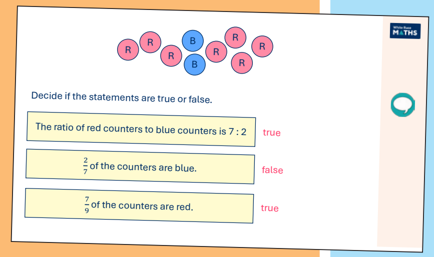
Premium supporting materials

Worksheets to accompany every small step, providing relevant practice questions for each topic that will reinforce learning at every stage.

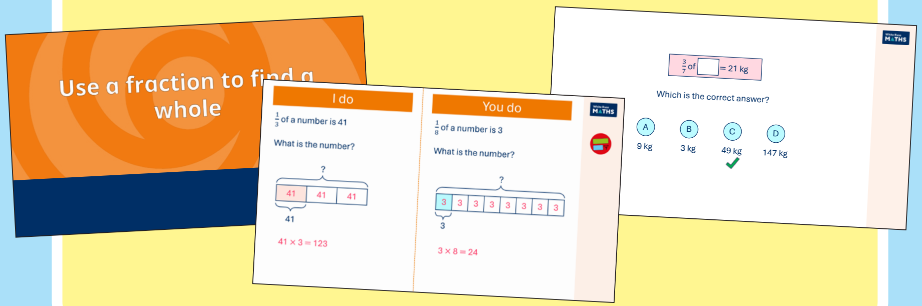
Answers to all the worksheet questions are provided.

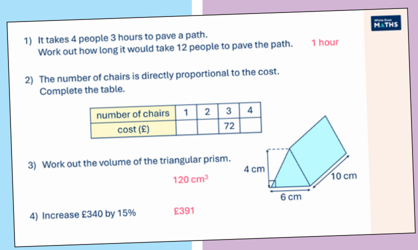


A **true or false** question for every small step in the scheme of learning. These can be used to support new learning or as another tool for revisiting knowledge at a later date.



Teaching slides for every small step, providing worked examples, multiple choice questions and open-ended questions. These are fully animated and editable, so can be adapted to the needs of any class.





Flashback 4 starter activities to improve retention.
Q1 is from the last lesson;
Q2 is from last week;
Q3 is from 2 to 3 weeks ago;
Q4 is from last term/year.

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Yearly overview

The yearly overview provides suggested timings for each block of learning, which can be adapted to suit different term dates or other requirements.

	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10	Week 11	Week 12
Autumn	Algebra **Algebraic manipulation**		Algebra **Equations, inequalities and formulae**		Algebra **Quadratic expressions and equations**		Number **Percentages**		Ratio, proportion and rates of change **Ratio and scale**		Number **Work with fractions**	
Spring	Number **Non-calculator methods**		Algebra **Straight line graphs**		Probability **Probability**		Number **Rounding and estimation**	Geometry and measures **Perimeter, area and volume**		Statistics **Interpret and represent data**		Algebra **Non-linear graphs**
Summer	Geometry and measures **Angles**		Statistics **Graphs and diagrams**		Geometry and measures **Vectors**		Number **Factors and powers**		Geometry and measures **Pythagoras' theorem and trigonometry**		Algebra **Simultaneous equations**	

White Rose MATHS

Autumn Block 1

Algebraic manipulation

Small steps

Step 1	Substitution
Step 2	Collect like terms
Step 3	Simplify expressions
Step 4	Addition and subtraction laws for indices
Step 5	Powers of powers
Step 6	Expand a single bracket
Step 7	Factorise into a single bracket

F Substitution

Notes and guidance

In this small step, students consolidate their understanding and application of substituting given values into algebraic expressions and formulae.

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Ensure that students practise substitution with and without a calculator. When using a calculator, highlight that, for example evaluating $3a^2$ when $a = 4$, can be calculated by typing 3×4^2 or $3(4)^2$

Mathematical talk

- What is meant by substitution?
- What does evaluate mean?
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- "The greater the substituted value, the greater the value of the expression." Give an example where this is not true.
- "If x is negative then the value of an expression containing x will be negative." Explain why this is not always true.
- If b is negative, explain why $10 - 2b$ will always be positive.
- What value of g cannot be substituted into $\frac{g}{(g - 3)}$? Explain your answer.

Misconceptions and common errors

- Students may incorrectly interpret expressions. For example, thinking $3a^2$ is equivalent to $(3a)^2$
- Students may incorrectly substitute negative numbers, especially when there is a negative coefficient of the variable. For example, substituting $a = -3$ into $7 - 2a$.

National Curriculum links

- Substitute numerical values into formulae and expressions, including scientific formulae (KS3)

F Substitution

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Teaching approaches

- Display cards showing expressions and values.

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  Ask students questions to support them with the substitution.

  - Which expressions require a division before the subtraction?
  - Which expressions will be positive?
    Which will be negative?

  Repeat with different expressions and values.

- Show students a formula.

  A rugby union team scores T points in a match.

  $$T = 7c + 5u + 3p$$

  c = number of converted tries scored

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  Ask students questions about the formula.

  - What is the value of T if $c = 2$, $u = 1$, $p = 5$?
  - If $T = 15$, what could c, u and p be?

Key vocabulary

expression collection of terms involving variables (letters) and numbers

variable symbol, usually a letter, that can represent any value in mathematical expressions, identities and formulae

formula mathematical rule or equation that shows the relationship between different variables or quantities

substitute replace letters with numerical values

Links and next steps

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- Support Curriculum – Year 8 Spring Block 5 – Step 4 – Substitution
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- Being fluent in substitution is an essential technique and is a part of the process of solving a pair of linear simultaneous equations.

F Collect like terms

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Notes and guidance

In this small step, students identify like terms in expressions and simplify by adding or subtracting like terms. Students will recap their understanding of collecting like terms from Key Stage 3. Remind students that like terms include the same variables to the same power, for example, a and $3a$ are like terms but $3a$ and $2a^2$ are not.

Include examples of terms with a coefficient of 1 and discuss the convention of writing these as, for example, c rather than $1c$. Include terms with squared or multiple variables, for example, t^2 or $4ab$, and discuss the commutative properties of like terms, such as $3pq$ and $3qp$.

Encourage students to use the '≡' symbol to indicate that the original and simplified expressions are equivalent.

Mathematical talk

- What does the ≡ symbol mean?
- Which of the terms are like terms?
- ______ and ______ are like/unlike terms
- What is the coefficient of ______?
- "Terms that contain the same letter(s) are always like terms." Give an example of where this statement is not true.
- Is bc the same as cb? Explain how you know.
- What is the coefficient of d^2?
- "All expressions can be simplified." Explain why this statement is not true.

Misconceptions and common errors

- Students may mistake terms with the same variable but different power as like terms, for example, b^2 and b.
- Students may remove the variable when subtracting like terms, for example, $10a - a \equiv 10$
- Students may attempt to simplify all terms in the expression including unlike terms, for example, $3a + 4b + 2a + b \equiv 10ab$.

National Curriculum links

- Simplify and manipulate algebraic expressions to maintain equivalence by: collecting like terms, multiplying a single term over a bracket, taking out common factors, expanding products of two or more binomials (KS3)

F

Collect like terms

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Teaching approaches

- Display cards showing some different terms.

$3a$	$3b$	ab	$-2a$	a^2	$5a$

Ask students to discuss which of these terms are like terms and justify their reasoning.

- Show students an expression to be simplified.

$$4x + 2xy + 7x + 3yx$$

Ask students questions to develop their understanding.

- Which of the terms are like?
- Is xy different from yx?
- Why is x not like xy?
- What is the total number of x's?
- What is the total number of xy's?
- What would the final simplified expression be?

Repeat with other expressions, including terms with negative coefficients and higher powers.

Key vocabulary

like terms	terms with the same variable(s) and power(s)
unlike terms	terms with different variable(s) and power(s)
expression	collection of terms involving variables (letters) and numbers
coefficient	number in front of a variable indicating the multiple of the variable

Links and next steps

- Support Curriculum – Year 8 Autumn Block 5 – Step 1 – Collect like terms
- Main Curriculum – Year 7 Autumn Block 3 – Step 4 – Collect like terms
- Students will later collect like terms when expanding and simplifying binomials.
- Challenge students to add and subtract with fractions that include like terms, for example, $\frac{3b}{2} + \frac{4b}{2}$

F Simplify expressions

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Notes and guidance

In this small step, students simplify expressions using multiplication and division, building on their knowledge from Key Stage 3. Remind students that $7 \times c$ is equivalent to $7c$ and $\frac{c}{3}$ is equivalent to $c \div 3$

Students should practise simplifying expressions that involve a fraction or division, which will require them to, at times, identify common numerical and/or simple algebraic factors. Begin by ensuring students are confident in simplifying expressions such as $5 \times a \times a$ and $a \times b \div 2$ before progressing to more complex expressions.

Students should also be confident that $a \times a$ should be written as a^2. Students will look at laws of indices later within this block, and powers greater than 2 are not explored in this step. Remind students that the process of simplifying an expression maintains equivalence.

Mathematical talk

- "All divisions can be written as a fraction."
  Is this statement true or false? Explain your answer.
- Does the order of the letters in a term matter?
- "When multiplying/dividing terms, you only need to multiply/divide their coefficients."
  Is the statement true or false? Explain your answer.
- What does the ≡ symbol mean?
- Suggest a multiplication/division that is equivalent to ______ (for example, $12b$).
- What is the same and what is different about the terms $\frac{4a}{1}$ and $4a$?
- What is the same and what is different about the terms $\frac{1}{4a}$, $\frac{1}{4}a$ and $\frac{a}{4}$?

Misconceptions and common errors

- Students may interpret ab and ba as unlike terms.
- Students may not identify common factors in divisions, for example, $\frac{12a}{8}$ can be simplified to $\frac{3a}{2}$

National Curriculum links

- Simplify and manipulate algebraic expressions (including those involving surds) by: simplifying expressions involving sums, products and powers, including the laws of indices

F Simplify expressions

Teaching approaches

- Display a match-up activity to students.

$6x \times 3y$	$36xy$
$6x \times 3 \times 2y$	$18x^2y^2$
$6x \times 3x \times y^2$	$18xy$
$2y \times 3x \times 6y$	$36xy^2$

Ask students to match each expression to its simplified form. Encourage discussions about why $x \times y$ can be written as xy, and xy is equivalent to yx.

- Ask students to discuss how else they could write the expression.

$6x \div 4$

Ask students to determine the correct, fully simplified answer.

A	B	C	D
$\frac{6x}{4}$	$\frac{3x}{2}$	$3x \div 2$	$\frac{4}{6x}$

Key vocabulary

simplify write an expression in a simpler equivalent form

like terms terms with the same variable(s) and power(s)

unlike terms terms with different variable(s) or power(s)

coefficient number in front of a variable indicating the multiple of the variable

Links and next steps

- Support Curriculum – Year 8 Spring Block 5 – Step 2 – Simplify expressions
- Main Curriculum - Year 8 Autumn Block 4 – Step 3 – Simplify expressions
- Students will later simplify expressions when rearranging formulae.
- Challenge students to solve equations that require simplifying the expression first, for example, solve $3x + 5x = 16$

F

Addition and subtraction laws of indices

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Notes and guidance

In this small step, students recap the addition and subtraction laws of indices, building on learning from Year 8. Remind students that $a^x \times a^y \equiv a^{x+y}$ and $\frac{a^x}{a^y} \equiv a^{x-y}$. It is important for students to note that to apply these laws, the base must be the same.

Students should be familiar with questions involving both numerical bases, for example, $2^3 \times 2^5$ and algebraic bases, for example $x^4 \times x^7$

Students should initially start with expressions with a coefficient of 1, before moving to expressions with greater coefficients. As confidence develops, progress students on to more complex expressions, including multiple variables, for example $\frac{12x^5y}{4x^3}$

Misconceptions and common errors

- Students may multiply/divide the powers instead of adding/subtracting.
- Students may not check the base is the same before adding or subtracting powers.
- Students may multiply the base when it is numerical, for example, $2^3 \times 2^4 = 4^7$
- Students may ignore operations when negative powers are involved, for example, writing $m^5 \div m^{-2} \equiv m^3$ instead of m^7

Mathematical talk

- What is the difference between a base and an index?
- What calculation is a^6 equivalent to? For example, $a \times a \times a$...
- Describe the addition law of indices.
- Describe the subtraction law of indices.
- Explain why the indices are added when multiplying powers.
- Explain why the indices are subtracted when dividing powers.
- Explain why $k^4 \times k$ is equivalent to k^5
- Can indices be negative? Explain your reasoning.
- Suggest a multiplication that is equivalent to $12x^5$
- Suggest a division that is equivalent to $12x^5$

National Curriculum links

- Simplify and manipulate algebraic expressions (including those involving surds) by: simplifying expressions involving sums, products and powers, including the laws of indices

F Addition and subtraction laws of indices

White Rose MATHS

Teaching approaches

- Present students with a statement.

$$3^5 \times 3^2 = 3^7$$

Model to students by writing the calculation out in full, showing why the statement is correct. Colour coding or labelling can be useful to illustrate each part of the calculation.

$$3^5 \times 3^2$$

$$3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^7$$

Repeat with other expressions, including algebraic bases and divisions, for example, $d^6 \times d^3$ and $\frac{e^9}{e^7}$

- Ask students to determine whether the calculations are true or false, justifying their answers.

$p^7 \times p^3 \equiv p^{10}$	$j^{19} \div j^4 \times j^7 \equiv j^{22}$
$c^{12} \div c^3 \equiv c^4$	$\frac{h^4 \times h^5}{h^3 \times h^2} \equiv h^{14}$
$k^5 + k^4 \equiv k^9$	$\frac{15e^7}{3e^2} \equiv 12e^5$
$5y^2 \times 3y^4 \equiv 15y^6$	$(t^6 \times t^3) \div t^9 \equiv t^2$

Key vocabulary

power way to express repeated multiplication of a number using a base and an index

base number that is raised to a certain index, for example, 10^3 has a base of 10

index written as a small number to the right and above the base number, which indicates how many times the base has been multiplied by itself

indices plural of index

Links and next steps

- Support Curriculum – Year 8 Spring Block 5 – Step 1 – Understand index notation
- Main Curriculum – Year 8 Spring Block 4 – Step 3 – Addition law for indices
- Main Curriculum – Year 8 Spring Block 4 – Step 4 – Subtraction law for indices
- Students will later use the addition and subtraction law of indices when multiplying/dividing values in standard form.

F Powers of powers

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Notes and guidance

In this small step, students build on their understanding of the laws of indices and now look at powers of powers. Students may have covered this previously in Year 8 as an extend step.

Begin exploring examples such as $(x^3)^2$ and discussing how this represents "x^3 squared", which is equivalent to x^3 multiplied by x^3. This allows students to use their understanding of the addition law of indices to make connections to explain why $(x^3)^2 \equiv x^6$. Encourage students to generalise by repeating with other expressions.

Once students are confident, progress to expressions that include coefficients of the variable, for example $(3x^4)^2$. If appropriate, challenge students to consider examples such as $2^2 \times 4^5$, where they need to adapt a value, so they have the same base, for example $2^2 \times (2^2)^5$

Mathematical talk

- How can $(x^2)^3$ be written out as a full calculation?
- Is $(a^b)^c$ the same as, or different to, $(a^c)^b$? Explain how you know.
- Describe the law of indices when there is a power of another power.
- "$(2g^3)^4 \equiv 2g^{12}$". Is this expression correct? Why or why not?
- How can 4 be written as a power of 2?
- "8^2 can be written as $(2^3)^2$, which is also equivalent to 2^6". Is this statement true or false? Explain how you know.
- Suggest a power of a power that is equivalent to x^{12}
- Explain why $(2m^3)^4$ is not the same as $(2m^4)^3$

Misconceptions and common errors

- Students may use the wrong law of indices and add/subtract the powers.
- Students may not include the coefficient inside the bracket in their calculation, for example, writing $(2x^5)^3 \equiv 2x^{15}$

National Curriculum links

- Simplify and manipulate algebraic expressions (including those involving surds) by: simplifying expressions involving sums, products and powers, including the laws of indices

F Powers of powers

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Teaching approaches

- Display a problem to students.

$(y^4)^3$

Discuss with students what "cubing" something means, and encourage students to determine how else they could write the expression. Model this with students.

$$y^4 \times y^4 \times y^4 \equiv y^{12}$$

Repeat with other questions that include both algebraic and numerical bases, e.g. $(5^2)^7$ and $(x^9)^4$. Encourage students to generalise, to determine a more efficient method.

- Display a question and possible answers to students.

$(2c^5)^3$

$6c^{15}$ | $8c^{35}$ | $8c^{15}$ | $8c^8$

Ask students to reason which answer is correct and describe the mistake that has been made in the incorrect answers.

Key vocabulary

power way to express repeated multiplication of a number using a base and an index

base number that is raised to a certain index, for example, 10^3 has a base of 10

index written as a small number to the right and above the base number, which indicates how many times the base has been multiplied by itself

indices plural of index

Links and next steps

- Main Curriculum – Year 8 Spring Block 4 – Step 6 – Powers of powers (E)
- Students will later use powers of powers when calculating with standard form, for example $(7.6 \times 10^5)^2$
- Challenge students to work with powers of powers with fractional values, e.g. $\left(\frac{1}{2x^3}\right)^4$

F Expand a single bracket

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Notes and guidance

In this small step, students recap expanding a single bracket, building on their learning from Key Stage 3. Ensure students are confident in expanding brackets in the form $a(bx + c)$ and $ax(bx + c)$ including expressions with negative values of a, b and c.

Once students are confident, begin to introduce expressions including indices, for example, $3x(x^2 + 4)$. It may be necessary to recap the laws of indices and simplifying expressions from the previous steps with students. Algebra tiles can be used to support students' understanding, where necessary and appropriate.

Students will also recap expanding and simplifying single brackets in the form $a(x \pm b) \pm c(x \pm d)$ and $a \pm b(x \pm c)$. Introduce expressions that include both positive and negative coefficients of the variable. Use terminology such as "multiply out" and "expand" for command words in questions to align with wording from exam boards.

Misconceptions and common errors

- Students may not multiply all terms inside the bracket by the term outside. For example, writing $3(x + 2) \equiv 3x + 2$
- Students may incorrectly add or subtract when expanding and simplifying with negatives.

Mathematical talk

- What does expand mean when working with brackets?
- What is the same and what is different about expanding $3(a + b)$ and $3(2a + b)$?
- Explain why $5(a + 3) \equiv 5a + 15$
- What is the addition law of indices? Why might this be needed when expanding a single bracket?
- "$x^3(x + 6)$ cannot be expanded."
  Is the statement true or false? Explain your answer.
- "When expanding two single brackets, the expanded expression can be simplified."
  Give an example of where this is true and not true.
- Explain why $-(4x + 7)$ is equivalent to $-4x - 7$

National Curriculum links

- Simplify and manipulate algebraic expressions to maintain equivalence by: collecting like terms, multiplying a single term over a bracket, taking out common factors, expanding products of 2 or more binomials (KS3)
- Simplify and manipulate algebraic expressions (including those involving surds) by: simplifying expressions involving sums, products and powers, including the laws of indices

F Expand a single bracket

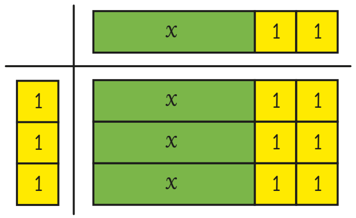
White Rose MATHS

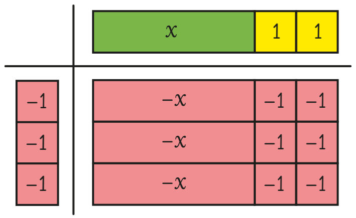
Teaching approaches

- Display three expressions.

A $3(x + 2)$ **B** $-3(x + 2)$ **C** $3x(x + 2)$

Ask students to discuss what is the same and what is different about each expression. Model expressions A and B using algebra tiles alongside a multiplication grid, then encourage students to consider how they could expand C.





	x	$+2$
$+3$	$3x$	$+6$

$3x + 6$

	x	$+2$
-3	$-3x$	-6

$-3x - 6$

After this, move on to explore expanding expressions with powers greater than 2, without the use of algebra tiles.

- Display an example of a student's answer to a question.

Expand and simplify $5(2x + 1) - 4(x - 2)$

$10x + 5 - 4x - 8 \equiv 6x - 3$

Ask students to determine the mistake that has been made, and write the correct simplified answer.

Key vocabulary

expand multiply each term inside the bracket by the expression outside the bracket

coefficient number in front of a variable indicating the multiple of the variable

expression collection of terms involving variables (letters) and numbers

Links and next steps

- Support curriculum – Year 9 Spring Block 7 – Step 1 – Expand single brackets and simplify (numerical coefficients)
- Support curriculum – Year 9 Spring Block 7 – Step 2 - Expand single brackets (algebraic coefficients)
- Main curriculum – Year 9 Spring Block 6 – Step 1 – Expand single brackets and simplify
- Students will expand and simplify when constructing algebraic arguments.
- Challenge students to expand a single bracket with fractional coefficients, for example, $\frac{3}{4}(5a + 6)$

F

Factorise into a single bracket

White Rose MATHS

Notes and guidance

In this small step, students recap factorising expressions into single brackets, building on their learning from Key Stage 3. It can be useful to revisit finding the highest common factor of a variety of terms, including those where the highest common factor is algebraic. Highlight to students that factorising and expanding are the inverse of each other. Using an area model or multiplication grid can be an excellent way to exemplify this, allowing students to make connections between the two. Students should explore factorising expressions including negative values and non-unit coefficients of the variable, including examples such as $6 - 2x$ and $-4x - 8$

Students' previous understanding of factorising is extended in this step, to include the subtraction law of indices, by including variables with powers greater than 2. For example, $2x^3 + 4x$.

Misconceptions and common errors

- Students may not fully factorise an expression by failing to identify the highest common factor.
- Students may correctly factorise the first part of an expression but not the other part(s). For example, writing $4y - 8 \equiv 4(y - 8)$
- Students may incorrectly divide terms with powers and divide indices, instead of applying the subtraction law.

Mathematical talk

- What is the highest common factor of ______ and ______?
  E.g. 30 and 12, x^2 and x or $8ab$ and $16b$
- What is the link between expanding a single bracket and factorising into a single bracket?
- Can the expression be factorised in more than one way?
- How do you know when an expression has been fully factorised? Explain your answer.
- Is it useful to identify 1 as a common factor?
  Explain your answer.
- "All expressions can be factorised."
  Give an example to show that this statement is false.
- Explain how you can check that you have factorised correctly.
  How does this link to expanding brackets?

National Curriculum links

- Simplify and manipulate algebraic expressions to maintain equivalence by: collecting like terms, multiplying a single term over a bracket, taking out common factors, expanding products of 2 or more binomials (KS3)
- Simplify and manipulate algebraic expressions (including those involving surds) by: simplifying expressions involving sums, products and powers, including the laws of indices

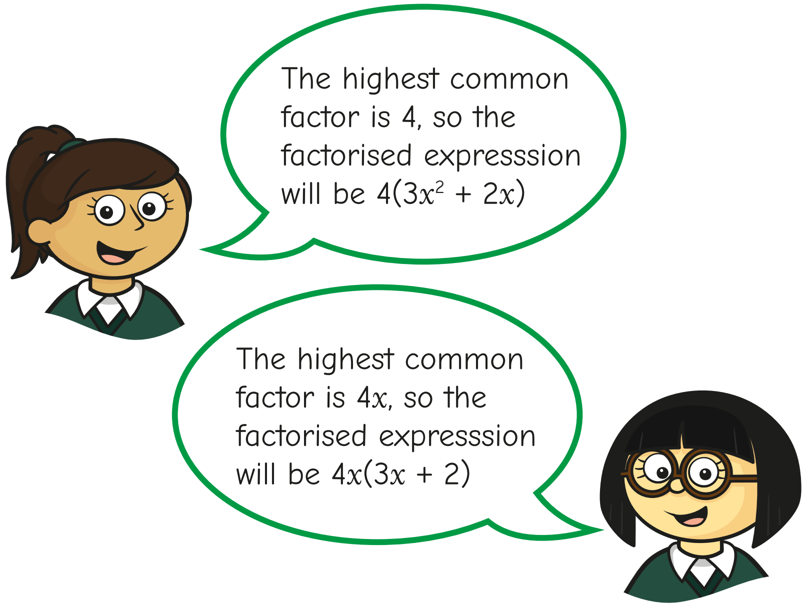
F Factorise into a single bracket

White Rose MATHS

Teaching approaches

- Display a question and answers given by two students.

Factorise $12x^2 + 8x$



Ask students to discuss who they agree with and encourage discussions with students about how both expressions are factorised but only one of them is fully factorised.

Repeat this by displaying more expressions to students but introducing negative values, multiple variables, and powers.

Key vocabulary

factorise	express an algebraic expression as a product of its factors using brackets, the reverse process of expanding brackets
coefficient	number in front of a variable indicating the multiple of the variable
highest common factor (HCF)	common factor of two or more integers or terms that has the greatest value or power
index	written as a small number to the right and above the base number, which indicates how many times the base has been multiplied by itself

Links and next steps

- Support curriculum – Year 8 Autumn Block 5 – Step 3 – Factorise into a single bracket
- Main curriculum – Year 9 Spring Block 6 – Step 2 – Factorise into a single bracket
- Students will later factorise quadratic expressions.
- Challenge students to factorise expressions with more than two terms such as $8x^2 + 12xy - 16x^2y$.

Autumn Block 2

Equations, inequalities and formulae

White Rose MATHS

Small steps

Step 1	Solve equations
Step 2	Solve fractional equations
Step 3	Solve equations with unknowns on both sides
Step 4	Understand inequalities
Step 5	Solve inequalities
Step 6	Change the subject of a simple formula
Step 7	Change the subject of a known formula
Step 8	Change the subject of a complex formula (E)

F

Solve equations

Notes and guidance

In this small step, students revisit solving 1- and 2-step equations, which were covered in Key Stage 3, before moving on to more complex equations later in the block. Representations such as bar models or function machines can be useful to highlight inverse operations. Students should be given opportunities to solve equations in different forms, for example, $17 = 5 + 2y$, as well as equations involving brackets and where solutions are non-integer and/or negative. Students will also solve equations where the unknown has a negative coefficient, such as $20 – 2y = 15$

Consider using questions where students do not rely on "spotting" the answer, as this can limit their understanding when they encounter more-complex equations. Calculators can be used if required. Students should also be encouraged to use substitution to check solutions.

Misconceptions and common errors

- Students may apply inverse operations incorrectly.
- Students may think all solutions have to be positive and/or integers.
- Students may struggle to solve equations presented in forms different to $ax + b = c$, such as $28 = 17 + w$
- Students may misunderstand equations like $23 – y = 15$

Mathematical talk

- What is different about an expression and an equation?
- What does $2a$ mean?
- If $2a + 1 = 10$, what is $2a$ equal to? Explain your answer.
- Is $2x + 8 = 18$ the same as $18 = 2x + 8$? How do you know?
- Explain why $x = 5$ and $5 = x$ are the same.
- How could we check if the solution is correct?
- "To solve a 2-step equation, you must always add or subtract something first."
  Is the statement true or false? Explain how you know.
- What is the inverse of ______?

National Curriculum links

- Recognise and use relationships between operations including inverse operations (KS3)
- Use and interpret algebraic notation, including: $3y$ in place of $y + y + y$ and $3 \times y$, and brackets (KS3)
- Simplify and manipulate algebraic expressions to maintain equivalence by: multiplying a single term over a bracket (KS3)
- Use algebraic methods to solve linear equations in one variable (including all forms that require rearrangement) (KS3)

F Solve equations

White Rose MATHS

Teaching approaches

- Present students with two equations.

$2w - 15 = 7$	$2w + 15 = 7$

Ask students questions about the equations.

- What is the same about the equations? What is different?
- What calculations are needed to solve the equations?
- How can you check the solutions?

Model the process of finding and checking the solutions.

$2w - 15 = 7$ (+ 15 on both sides)
$2w = 22$ (÷ 2 on both sides)
$w = 11$

$2w - 15 = 7$ (– 15 on both sides)
$2w = -8$ (÷ 2 on both sides)
$w = -4$

When students are confident, ask them to solve other equations involving brackets, directed numbers or non-integer solutions.

Key vocabulary

equation statement to show that two expressions are equal

inverse opposite effect of

expression collection of terms involving variables and numbers

unknown value not yet known, indicated with a letter and used in an equation or inequality

solution value of an unknown that satisfies an equation or inequality

Links and next steps

- Students will substitute into formulae such as $P = I^2R$ and solve the equation to find an unknown value.
- Main curriculum – Year 8 Spring Block 2 – Step 1 – Solve 1- and 2-step equations
- Support curriculum – Year 9 Autumn Block 4 – Step 2 – Solve equations with brackets
- Students will later solve 2-step equations involving squares or square roots, such as $a^2 + 5 = 30$ or $\sqrt{b} - 1 = 10$

F

Solve fractional equations

White Rose MATHS

Notes and guidance

In this small step, students revisit solving fractional equations, consolidating learning from Key Stage 3. Function machines can be a useful representation to highlight key differences between equations such as $\frac{x}{5} + 1 = 13$ and $\frac{x+1}{5} = 13$

Encourage students to discuss the order in which inverse operations should be applied, and ensure that practice includes a mixture of different forms of these equations, to avoid reliance on repetitive procedures. As students' confidence develops, more complex equations can be introduced, such as $\frac{2x+6}{3} = 8$

Practice should include equations with non-integer or negative solutions.

Mathematical talk

- Which operations are used in the equation?
- The inverse of ______ is ______
- What is the same and what is different about the equations $\frac{x}{2} + 1 = 6$ and $\frac{x+1}{2} = 6$?
- To solve an equation with fractions, is the first step always adding or subtracting a number? Explain how you know.
- To solve an equation with a bracket, do you have to expand the bracket first? Explain how you know.

Misconceptions and common errors

- Students may apply inverse operations incorrectly or in the wrong order, thinking that, for example, $\frac{x+1}{3} = 5$ can be simplified to $\frac{x}{3} = 4$ by subtracting 1 from both sides.
- Students may struggle to solve equations presented in different forms such as $4 = \frac{2+6x}{5}$
- Students may make arithmetic errors when solving equations with negative solutions or coefficients.

National Curriculum links

- Recognise and use relationships between operations including inverse operations (KS3)
- Use and interpret algebraic notation, including: $3y$ in place of $y + y + y$ and $3 \times y$, brackets and $\frac{a}{b}$ in place of $a \div b$ (KS3)
- Simplify and manipulate algebraic expressions to maintain equivalence by: multiplying a single term over a bracket (KS3)
- Use algebraic methods to solve linear equations in one variable (including all forms that require rearrangement) (KS3)

F Solve fractional equations

White Rose MATHS

Teaching approaches

- Show students two similar equations with different structures.

$\frac{x}{3} + 5 = 9$ $\quad\quad$ $\frac{x + 5}{3} = 9$

Ask students to discuss what is the same and what is different about the equations, highlighting how the order of operations are different.

Next, model using a function machine to solve the equations.

input x → $\div 3$ → $+ 5$ → 10 output

input x → $+ 5$ → $\div 3$ → 9 output

Ask students questions to check understanding.

- What is the same and what is different about the function machines?
- How can you work out the value of the input for each function machine?

Repeat with other equations, progressing to equations that have non-integer and/or negative solutions.

Key vocabulary

equation statement to show that two expressions are equal

unknown value not yet known, indicated with a letter and used in an equation or inequality

solution value of an unknown that satisfies an equation or inequality

Links and next steps

- Students will substitute values into formulae such as $E_k = \frac{1}{2}mv^2$, solving to work out the value of an unknown.
- Main curriculum – Year 8 Spring Block 2 – Step 3 – Solve fractional equations
- Support curriculum – Year 8 Spring Block 2 – Step 5 – Solve fractional equations
- Challenge students to solve fractional equations involving squares or square roots, such as $\frac{a^2 + 1}{2} = 5$

F

Solve equations with unknowns on both sides

White Rose MATHS

Notes and guidance

In this small step, students solve equations with unknowns on both sides of the equals sign. This may be a new concept for some students, as equations of this form are not addressed in the Key Stage 3 Support curriculum. Using bar models alongside abstract calculations is a useful strategy to support students.

Encourage students to identify which of the two coefficients of the unknown is the smallest, ensuring students see examples of equations with the greater coefficient of an unknown on the right side as well as the left side. Practice should include both non-integer and negative solutions. Equations of the form $ax + b = cx$ should also be explored. If appropriate, students could also solve equations involving negative coefficients of an unknown, such as $20 - x = 4x + 5$

Encourage students to check solutions by using substitution.

Misconceptions and common errors

- Students may confuse unknowns with constants, such as subtracting 3 instead of $3x$.
- Students may always subtract unknowns from both sides, even when adding is more efficient.
- Students may "lose" a negative sign when solving an equation.

Mathematical talk

- What is the value of the coefficient?
- What is the inverse of adding $2x$?
- What is the same and what is different about $9 + 3x = 2x + 4$ and $9 - 3x = 2x + 4$?
- What could the first step to solve the equation be?
- Do you always add or subtract a constant as a first step to solve an equation with unknowns on both sides? Explain how you know.
- Is $t = 5$ the same as $5 = t$? Explain your answer.
- How can you check the solution?

National Curriculum links

- Recognise and use relationships between operations including inverse operations (KS3)
- Use and interpret algebraic notation, including: $3y$ in place of $y + y + y$ and $3 \times y$, and brackets (KS3)
- Simplify and manipulate algebraic expressions to maintain equivalence by: multiplying a single term over a bracket (KS3)
- Use algebraic methods to solve linear equations in one variable (including all forms that require rearrangement) (KS3)

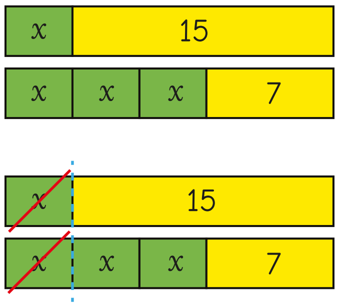
F

Solve equations with unknowns on both sides

White Rose MATHS

Teaching approaches

- Model how to use a bar model to solve an equation with unknowns on both sides.



$x + 15 = 3x + 7$ ($-x$)

$15 = 2x + 7$ (-7)

$8 = 2x$ ($\div 2$)

$4 = x$

- Show students two similar equations.

$26 + m = 3m + 2$ ($-m$)	$26 - m = 3m + 2$ ($+m$)
$26 = 2m + 2$ (-2)	$26 = 4m + 2$ (-2)
$24 = 2m$ ($\div 2$)	$24 = 4m$ ($\div 4$)
$12 = m$	$6 = m$

Ask students to discuss what is the same and what different about the equations.

Model how to solve them, highlighting key decisions such as why the first step for the second equation is adding m to both sides.

Key vocabulary

equation statement to show that two expressions are equal

unknown value not yet known, indicated with a letter and used in an equation or inequality

solution value of an unknown that satisfies an equation or inequality

coefficient number in front of a variable indicating the multiple of the variable

Links and next steps

- Main curriculum – Year 9 Autumn Block 4 – Step 4 – Solve equations and inequalities with unknowns on both sides
- Students will later form and solve equations with unknowns on both sides.
- Challenge students to solve more complex fractional equations such as $\frac{2(x-5)}{1-3x} = 2$

F

Understand inequalities

White Rose MATHS

Notes and guidance

In this small step, students interpret and use inequalities, building on prior learning from Key Stage 3. Begin by reviewing the meaning of each inequality symbol, highlighting key concepts such as the difference between $x < 3$ and $x \leq 3$

Students should also understand inequalities where the unknown is on the right-hand side, appreciating that, for example, $m < 8$ is equivalent to $8 > m$. Encourage discussions around integer solutions that satisfy a given inequality and introduce compound inequalities such as $3 < y \leq 7$

When working with number lines, ensure that students understand the associated notation, such as which inequality symbols correspond to shaded or unshaded circles.

Misconceptions and common errors

- Students may misinterpret inequalities symbols. For example, interpreting $x \leq 2$ as "x is less than 2".
- Students may struggle to interpret inequalities where the variable is on the right-hand side, such as $5 < y$.
- Students may think that inequalities may only have integer solutions.
- Students may incorrectly represent inequalities on a number line, such as using a shaded instead of an unshaded circle.

Mathematical talk

- How can you describe the inequality in words?
- Which values will/will not satisfy the inequality?
- What is the greatest/smallest integer value that satisfies the inequality?
- Are all the solutions integers? Explain your answer.
- What is the same and what is different about $a > 4$ and $a \geq 4$?
- What is the same and what is different about $b < 5$ and $5 > b$?
- Should the representation use a shaded or unshaded circle?
- Which direction should the arrow be pointing on the number line? How do you know?

National Curriculum links

- Order positive and negative integers, decimals and fractions; use the number line as a model for ordering of the real numbers; use the symbols =, ≠, <, >, ≤, ≥ (KS3)
- Understand and use the concepts and vocabulary of expressions, equations, inequalities, terms and factors (KS3)

F

Understand inequalities

White Rose MATHS

Teaching approaches

- Display some inequalities.

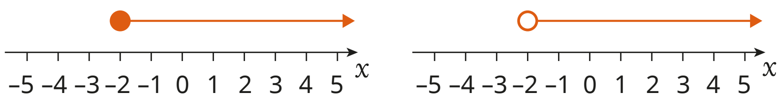
$t > 6$ $t \geq 6$

Ask students questions to deepen their understanding.

- What is the same and what is different?
- Which of the inequalities does 6 satisfy?
- What is the smallest integer value that satisfies each inequality?
- Write another inequality that 6 will/will not satisfy.

- Show students two inequalities represented on number lines.

$x \geq -2$ $x > -2$



Ask students questions about the inequalities.

- Which of the inequalities includes the number –2?
- What does a shaded/unshaded circle represent?
- How could you represent $x \leq -2$ or $x < -2$?

Key vocabulary

integer	whole number that can be positive or negative
inequality	mathematical statement that shows the relationship between two expressions where one is greater than, less than, or not equal to the other
satisfy	find a value that makes the inequality true when substituted into the inequality
compound inequality	inequality that combines two simple inequalities, such as $2 < g < 5$

Links and next steps

- Support curriculum – Year 9 Autumn Block 4 – Step 3 – Understand inequalities
- Main curriculum – Year 8 Spring Block 2 – Step 6 – Understand and use inequalities
- Students will later form and solve inequalities.

F

Solve inequalities

White Rose MATHS

Notes and guidance

In this small step, students will recap solving linear inequalities. It is useful to highlight similarities between equations and inequalities, as the strategies to solve both are similar. Students need to understand a linear equation has one solution, whereas a linear inequality has a range of solutions.

Make links to the previous step, discussing possible integer solutions and provide opportunities for students to represent these on a number line. If appropriate, students can also solve inequalities with unknowns on both sides. Be mindful that some students may not have been introduced to inequalities of this form, as this is not covered in the Key Stage 3 support curriculum.

Students will look at solving compound inequalities in Year 11

Misconceptions and common errors

- Students may give solutions using an "=" symbol, for example stating $x = 2$ rather than $x < 2$
- Students may misinterpret inequalities with an unknown on the right side, such as interpreting $5 < x$ as "x is less than 5".
- Students may represent inequalities incorrectly on a number line, such as using a shaded circle rather than an unshaded circle.

Mathematical talk

- What is the same and what is different about an equation and an inequality?
- How is $x = -3$ different to $x \geq -3$?
- Is the method to solve an equation the same as solving an inequality? Explain your answer.
- What is the smallest/greatest possible integer solution of the inequality?
- Explain why $5 < x$ is the same as $x > 5$
- What does it mean for a solution to "satisfy" an inequality?
- How can you check your solutions?
- Which values would be useful to check the solution set is correct?

National Curriculum links

- Order positive and negative integers, decimals and fractions; use the number line as a model for ordering of the real numbers; use the symbols $=, \neq, <, >, \leq, \geq$ (KS3)
- Understand and use the concepts and vocabulary of expressions, equations, inequalities, terms and factors (KS3)

F

Solve inequalities

White Rose MATHS

Teaching approaches

- Model strategies to solve an inequality.

$2x + 7 \leq 19$

-7 -7

$2x \leq 12$

$\div 2$ $\div 2$

$x \leq 6$

Ask students questions to deepen their understanding.

- Is 6 a solution? How do you know?
- Is 7 a solution? How do you know?
- How many solutions are there?
- How would you describe the solutions to the inequality?
- What is the same and what is different about the solutions to $2x + 7 \leq 19$ and $2x + 7 = 19$?
- What would the solutions to $2x + 7 \geq 19$ be?
- What would the solutions to $2x + 7 < 19$ be?
- What would the solutions to $19 \geq 2x + 7$ be?

Key vocabulary

inequality – mathematical statement that shows the relationship between two expressions where one is greater than, less than, or not equal to the other

satisfy – find a value that makes the inequality true when substituted into the inequality

integer – whole number that can be positive or negative

solve – find the value(s) or set of values of an unknown in an equation or inequality

Links and next steps

- Main curriculum – Year 8 Spring Block 2 – Step 8 – Solve inequalities
- Main curriculum – Year 8 Spring Block 2 – Step 10 – Solve inequalities with unknowns on both sides (E)
- Support curriculum – Year 9 Autumn Block 4 – Step 5 – Solve 1-step inequalities
- Support curriculum – Year 9 Autumn Block 4 – Step 6 – Solve 2-step inequalities

F

Change the subject of a simple formula

White Rose MATHS

Notes and guidance

In this small step, students rearrange simple formulae that involve one or two steps of working, to change the subject. Note that this may be the first time students have changed the subject of a formula, as this is not covered in the Key Stage 3 Support curriculum. Bar models and function machines can be useful representations to support students' understanding.

Drawing comparisons to solving equations is useful as students begin to explore this concept. Ensure that students understand the subject of a formula is the variable that is isolated and equal to an expression involving other variables.

Discuss the fact that the subject of a formula can appear on either the left or right side of the equals sign. For example, r is the subject of both $r = m + p$ and $m + p = r$.

Misconceptions and common errors

- Students may think formulae such as $p = y + 3x^2$ require more steps of working to rearrange to make y the subject than, for example, the formula $p = y + x$.
- Students may incorrectly rearrange formulae with fractions, such as subtracting 3 as a first step to rearrange $x = \frac{y + 3}{5}$
- Students may incorrectly multiply terms by an unknown or a constant, such as writing $a \times (x + y) = ax + y$.

Mathematical talk

- Which variable is the subject of the formula? How do you know?
- Can a formula be rearranged to make any variable the subject? Explain your answer.
- What is the inverse operation of ______?
- Should the subject of a formula always be on the left of the equals sign? Explain your answer.
- Can the subject of a formula be a numerical value? Explain how you know.
- Explain why $a + b$ multiplied by c is not equal to $ac + b$.

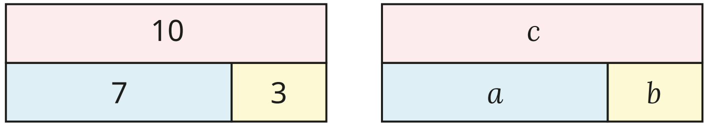
National Curriculum links

- Understand and use standard mathematical formulae; rearrange formulae to change the subject (KS3)
- Recognise and use relationships between operations including inverse operations (KS3)

F Change the subject of a simple formula

Teaching approaches

- Show students two bar models.

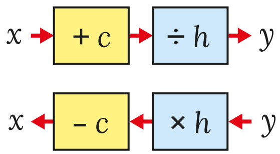


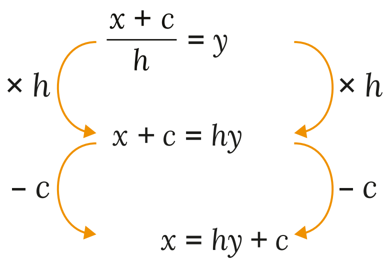
$7 + 3 = 10$
$3 + 7 = 10$
$10 - 3 = 7$
$10 - 7 = 3$

$a + b = c$
$b + a = c$
$c - b = a$
$c - a = b$

Ask students to write a fact family for the first bar model, then repeat for the second bar model. Highlight that c is the subject of the formula $a + b = c$, that b is the subject of $c - a = b$ and that a is the subject of the formula $c - b = a$. Encourage students to discuss what they think "the subject of a formula" means.

- Model how to change the subject of a formula with a function machine.





Key vocabulary

formula mathematical rule or equation that shows the relationship between different variables or quantities

subject variable that is isolated on one side of the formula

rearrange adjust a formula to isolate a specific variable or simplify an expression

Links and next steps

- Students may manipulate formulae to solve for variables, usually after substituting in given values.
- Main curriculum – Year 9 Autumn Block 4 – Step 7 – Change the subject of a formula (one-step)
- Main curriculum – Year 9 Autumn Block 4 – Step 8 – Change the subject of a formula (two-step)
- Students will rearrange formulae to solve problems in other areas of maths, such as rearranging $A = \pi r^2$ to calculate the radius of a circle when given the area.

F

Change the subject of a known formula

White Rose MATHS

Notes and guidance

In this small step, students rearrange known formulae, building on learning from Key Stage 3. Students use rearrangements of formulae to solve problems, such as making d the subject of $C = \pi d$ to work out the diameter of a circle when given the circumference. This provides students with a context of why changing the subject of a formula is a useful skill.

If appropriate, students could be asked to recall known formulae rather than having formulae provided. Rewriting formulae involving fractions may be useful to highlight key steps when rearranging. For example, writing $A = \frac{1}{2}bh$ as $A = \frac{bh}{2}$ to highlight that both sides of the formula can be multiplied by 2 as a first step to make b or h the subject.

Mathematical talk

- What is the subject of the formula?
- How can we make ______ the subject of the formula?
- How is changing the subject of a formula similar to solving an equation?
- Why is changing the subject of a formula useful?
- Can you change the subject of any formula?
- What is the same and what is different about the formulae $A = \frac{1}{2}bh$ and $A = \frac{bh}{2}$?

Misconceptions and common errors

- Students may struggle to comprehend a formula when the subject appears on the right-hand side of the equals sign. For example, $\frac{2A}{h} = b$
- Students may struggle to rearrange formulae involving fractions or brackets. For example, $A = \frac{1}{2}bh$ or $A = \frac{1}{2}(a + b)h$.

National Curriculum links

- Understand and use standard mathematical formulae; rearrange formulae to change the subject (KS3)
- Recognise and use relationships between operations including inverse operations (KS3)

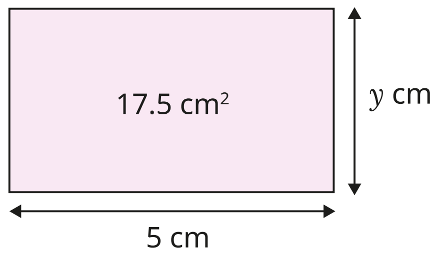
F

Change the subject of a known formula

White Rose MATHS

Teaching approaches

- Provide students with some information related to a rectangle.



$\div l$ $\quad A = lw \quad$ $\div l$

$\frac{A}{l} = w$

$\frac{17.5}{5} = y$ (substitute)

$3.5 = y$ (solve)

Model how to rearrange the formula used to work out the unknown length.

- Show students different rearrangements of a formula and ask them to discuss which are correct.

$A = \frac{1}{2}(a + b)h$

$2A = (a + b)h$

$2Ah = a + b$

$\frac{2A}{h} + b = a$

$2A - ah = bh$

Key vocabulary

formula mathematical rule or equation that shows the relationship between different variables or quantities.

subject variable that is isolated on one side of the formula

variable symbol, usually a letter, that can represent any value in mathematical expressions, identities and formulae

rearrange adjust a formula to isolate a specific variable or simplify an expression

Links and next steps

- Students may manipulate formulae to solve for variables, usually after substituting in given values.
- Main curriculum – Year 9 Autumn Block 4 – Step 8 – Change the subject of a formula (two step)
- Students will rearrange formulae to solve problems in other areas of maths, such as rearranging $A = \pi r^2$ to calculate the radius of a circle when given the area.

F Change the subject of a complex formula E

White Rose MATHS

Notes and guidance

In this small step, students rearrange complex formulae involving combinations of squares, square roots, fractions and brackets. Explore strategies to rearrange formulae where the proposed subject has a negative coefficient, such as making t the subject of $y = x - t$. Comparing this to solving a "subtract from" equation such as $5 = 12 - t$ could be useful.

Students could then move on to formulae like this involving fractions. Once students are confident, introduce formulae involving brackets, while comparing different strategies, such as expanding a bracket or dividing by the coefficient of the bracket.

Students should also be given opportunities to rearrange formulae with squared terms and/or respective roots.

Misconceptions and common errors

- Students may incorrectly rearrange when the proposed subject has a negative coefficient. For example, rearranging $a - x = b$ to $-x = b - a$ to make x the subject.
- Students may apply inverse operations incorrectly. For example, being given $a + b = c^2$ and stating $c = \sqrt{a} + \sqrt{b}$
- Students may square terms with coefficients incorrectly, for example, writing $(5y)^2$ as $5y^2$ instead of $25y^2$

Mathematical talk

- What is the subject of the formula?
- Which operations are applied to x in the expression $\frac{ax + b}{c}$? In what order are they applied?
- Is the coefficient positive or negative? Explain your answer.
- What is the same and what is different about making x the subject of $y = x + a$ and $y = x + 2a$?
- How could you check that your rearrangement is correct?
- The inverse of ______ is ______
- "If $y = x^2$, then $x = \sqrt{y}$."
  Is the statement true or false? Explain your answer.
- "t is the subject of $-t = 3 + h$."
  Is the statement true or false? Explain your answer.

National Curriculum links

- Understand and use standard mathematical formulae; rearrange formulae to change the subject (KS3)
- Recognise and use relationships between operations including inverse operations (KS3)

F Change the subject of a complex formula E

White Rose MATHS

Teaching approaches

- Present a formula where the proposed subject has a negative coefficient.

$y = x - 2a$

Make a the subject of the formula.

$$y = x - 2a$$
$$(+2a) \quad y + 2a = x \quad (+2a)$$
$$(-y) \quad 2a = x - y \quad (-y)$$
$$(\div 2) \quad a = \frac{x - y}{2} \quad (\div 2)$$

- Demonstrate different strategies to rearrange a formula with a bracket.

$$m = 2(k + t)$$
$$\text{expand} \quad m = 2k + 2t$$
$$(-2k) \quad m - 2k = 2t \quad (-2k)$$
$$(\div 2) \quad \frac{m - 2k}{2} = t \quad (\div 2)$$

$$m = 2(k + t)$$
$$(\div 2) \quad \frac{m}{2} = k + t \quad (\div 2)$$
$$(-2) \quad \frac{m}{2} - k = t \quad (-2)$$

Students could be encouraged to substitute values to show that both of the rearranged formulae are equivalent.

Key vocabulary

formula	mathematical rule or equation that shows the relationship between different variables or quantities.
subject	variable that is isolated on one side of the formula
rearrange	adjust a formula to isolate a specific variable or simplify an expression
coefficient	number in front of a variable indicating the multiple of the variable

Links and next steps

- Students may manipulate formulae to solve for variables, usually after substituting in given values.
- Main curriculum – Year 9 Autumn Block 4 – Step 9 – Change the subject of complex formulae (E)
- Students will rearrange formulae to solve problems with more-complex formulae, such as rearranging $V = \pi r^2 h$ to calculate the height of a cylinder when given the volume and radius.

Autumn Block 3

Quadratic expressions and equations

White Rose MATHS

Small steps

Step 1	Expand double brackets
Step 2	Factorise quadratic expressions (positive only)
Step 3	Factorise quadratic expressions
Step 4	Difference of two squares (E)
Step 5	Solve quadratic equations equal to 0
Step 6	Solve quadratic equations by factorisation
Step 7	Quadratic graphs of the form $y = x^2 + a$

F Expand double brackets

White Rose MATHS

Notes and guidance

In this small step, students will build on their knowledge of expanding double brackets in the form $(ax \pm b)(cx \pm d)$ from Key Stage 3. Begin with expressions in the form $(x + a)(x + b)$, with all positive terms, and introduce negative terms as students' confidence develops, before introducing expressions with coefficients of the variable. Expressions in unfamiliar forms such as $(x + 2)^2$ or $(x + 2)(3 + x)$ can also be explored.

Recap the use of an area model as a visual prompt for discussion on how to expand binomials, using algebra tiles to support if necessary. Students need to be confident with simplification and calculating with negative numbers. Where appropriate, extend students by expanding brackets in other mathematical contexts, for example, expressing the area of rectilinear shapes with binomial dimensions.

Misconceptions and common errors

- Students may only multiply terms together in corresponding positions, for example, $(x + 3)(x + 2) \equiv x^2 + 6$
- Students may correctly multiply the first three terms but add the fourth terms, for example, $(x + 3)(x + 2) \equiv x^2 + 3x + 2x + 5$
- In examples such as $(x + 4)^2$, students may square each term in the bracket to get $x^2 + 16$

Mathematical talk

- $(a + 6)(a - 4)$ means ______ multiplied by ______
- Can an expression always be simplified after expanding?
- "$(2x + 5)(4x + 3) \equiv 8x^2 + 6x + 20x + 8$"
  Is the statement true or false? Explain your answer.
- Do simplified quadratic expressions always have three terms?
- What is the same and what is different about expanding $(3 - x)(2 - x)$ and $(x - 3)(x - 2)$?
- How can you ensure all the terms from each bracket have been multiplied?
- How can you rewrite $(x + 3)^2$ in another form?

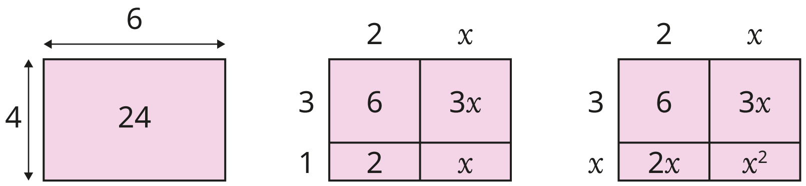
National Curriculum links

- Simplify and manipulate algebraic expressions to maintain equivalence by: collecting like terms, multiplying a single term over a bracket, taking out common factors, expanding products of two or more binomials (KS3)
- Simplify and manipulate algebraic expressions (including those involving surds) by: simplifying expressions involving sums, products and powers, including the laws of indices

F Expand double brackets

Teaching approaches

- Display three diagrams to students.

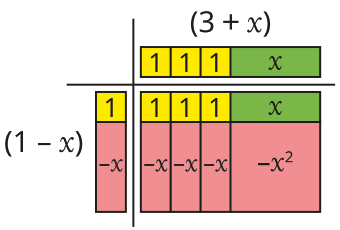


Ask students questions to deepen their understanding.

  - What is the same and what is different?
  - What multiplications are represented in each diagram?
  - What is the simplified answer to each calculation?

Repeat with similar expressions and introduce expressions with coefficients of the variable.

- Model to students using algebra tiles why $(1 - x)(3 + x) \equiv 3 - 2x - x^2$



	3	$+x$
1	3	$+x$
$-x$	$-3x$	$-x^2$

Link to a multiplication grid to allow students to make connections to previous examples. Remind students of zero pairs when simplifying their answers.

Key vocabulary

quadratic equation or expression involving the second (and no higher) power of a variable or unknown, typically in the form $ax^2 + bx + c$

expand multiply each term inside the bracket by the expression outside the bracket

term number or variable or any combination of number and variable combined by multiplication or division

Links and next steps

- Support Curriculum – Year 9 Spring Block 7 – Step 3 – Expand double brackets
- Main curriculum - Year 9 Spring Block 6 – Step 3 – Expand double brackets
- Students will later expand double brackets when solving algebraic area problems.
- Challenge students to expand brackets that include a trinomial, for example $(2a + b + c)(a - c)$.

F

Factorise quadratic expressions (positive only)

White Rose MATHS

Notes and guidance

In this small step, students will factorise quadratic expressions in the form $x^2 + bx + c$, which they may have previously explored in Key Stage 3. Students will look at factorising expressions including negatives in the following step.

Begin by asking students to find two numbers that make a specific sum and product, for example, find two numbers that have a sum of 7 and a product of 10

Support students to make connections by explicitly showing that factorising is the inverse of expanding brackets. Using algebra tiles and multiplication grids can strengthen students' conceptual understanding.

Misconceptions and common errors

- Students may think that, for example, $x^2 + 5x + 6$ factorises to $(x + 5)(x + 6)$.
- Students may write a pair of factors that multiply to give b and sum to give c.
- Students may correctly find a factor pair for c but forget to check if they sum to b.

Mathematical talk

- What are the factors of ______?
- Which pair of factors sum to ______?
- How do the factors of the constant term relate to the coefficient of x?
- If $x^2 + 6x + 5$ is the expanded expression, what was the factorised expression?
- How can you check your answer?
- Does the order of the brackets of a factorised expression matter? Explain your answer.
- Give an example of a quadratic expression that cannot be factorised.
- Why is factorising $x^2 + 4x + 3$ different from factorising $x^2 + 4x$?

National Curriculum links

- Simplify and manipulate algebraic expressions (including those involving surds) by: factorising quadratic expressions of the form $x^2 + bx + c$, including the difference of two squares

F Factorise quadratic expressions (positive only)

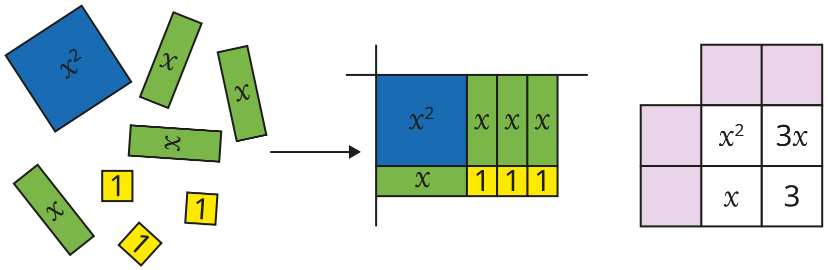
White Rose MATHS

Teaching approaches

- Display a quadratic expression to students.

$x^2 + 4x + 3$

Model to students how this expression can be represented using algebra tiles and a multiplication grid.



Ask students questions to deepen understanding.

- What do you notice about the arrangement of the algebra tiles?
- Why do you think the tiles have to be arranged in this way to factorise?
- What are the missing terms in the multiplication grid?
- What do you notice about the terms in the binomials and the initial quadratic expression?
- Can you make any generalisations?

Key vocabulary

sum result of adding two or more numbers together

product result of a multiplication

factorise express an algebraic expression as a product of its factors using brackets; the reverse process of expanding brackets

Links and next steps

- Main curriculum – Year 9 Spring Block 6 – Step 6 – Factorise quadratic expressions
- Students will later factorise quadratics when solving algebraic area problems.
- Challenge students to factorise expressions including multiple variables, for example, $x^2 + 2ax + a^2$

F

Factorise quadratic expressions

White Rose MATHS

Notes and guidance

In this small step, students will factorise quadratic expressions in the form $x^2 \pm bx \pm c$. This will build on students' understanding from the previous step of factorising expressions in the form $x^2 + bx + c$. In this step, it may be useful to recap calculations with directed numbers. Similarly to the last step, begin by asking students to find two numbers that make a specific sum and product, for example, find two numbers that have a sum of –6 and a product of 8, and encourage students to factorise an expression using these values, for example, $x^2 - 6x + 8$

Using algebra tiles and multiplication grids can strengthen students' conceptual understanding. Once students are confident, progress onto factorising expressions such as $x^2 + 7x - 30$ and $x^2 - x - 12$

Misconceptions and common errors

- Students may write a pair of factors that multiply to give b and sum to give c.
- Students may use the incorrect sign within a bracket. For example, factorising $x^2 + 2x - 3$ to $(x - 1)(x - 3)$.
- When asked to write an expression in the form $(x + a)(x + b)$, students may assume that a and b must be positive integers and not include negative integers.

Mathematical talk

- What are the factors of ______?
- Which pair of factors sum to ______?
- What are the factors of –12?
- Explain why –3 and –4 are a factor pair of 12
- What will the signs inside the brackets be when factorising $x^2 - 7x + 12$? Both plus signs, both minus signs or will they be different?
- When will both the signs inside the brackets be negative? Explain how you know.
- "$x^2 - 2x - 15$ will have a negative inside each bracket as there are two negative terms in the expression."
  Explain why this is not true.
- How can you check your factorised expression is correct?

National Curriculum links

- Simplify and manipulate algebraic expressions (including those involving surds) by: factorising quadratic expressions of the form $x^2 + bx + c$, including the difference of two squares

F Factorise quadratic expressions

White Rose MATHS

Teaching approaches

- Display two quadratic expressions to students.

A	B
$x^2 + 7x + 12$	$x^2 - 7x + 12$

  Ask students questions to deepen understanding.
  - What is the same and what is different?
  - What would expression A be when factorised?
  - How would expression B differ when factorised?
  - Why can the expression $x^2 - 7x - 12$ not be factorised?

- Display a match-up activity to students involving expressions that look similar but are different. This will help students to focus on what changes and what remains the same, deepening their understanding and generalisation of the concept.

$x^2 + 5x + 6$	$x^2 - 5x + 6$	$x^2 - 5x - 6$	$x^2 + 5x - 6$
$(x + 1)(x - 6)$	$(x + 2)(x + 3)$	$(x - 2)(x - 3)$	$(x + 6)(x - 1)$

Key vocabulary

variable symbol, usually a letter, that can represent any value in mathematical expressions, identities and formulae

expression collection of terms involving variables and numbers

quadratic equation or expression involving the second (and no higher) power of a variable or unknown, typically in the form $ax^2 + bx + c$

factorise express an algebraic expression as a product of its factors using brackets; the reverse process of expanding brackets

Links and next steps

- Main curriculum – Year 9 Spring Block 6 – Step 6 – Factorise quadratic expressions
- Students will later factorise quadratics when solving algebraic area problems.
- Challenge students to factorise expressions with a negative coefficient of the variable, for example, $-x^2 + x + 6$

F Difference of two squares E

White Rose MATHS

Notes and guidance

In this small step, students will explore expressions in the form $x^2 - a^2$ where a is an integer. Encourage students to expand brackets of the form $(x \pm a)(x \mp a)$ to uncover the difference of two squares and explore the connection to quadratics in the form $x^2 \pm bx \pm c$, noting how the middle term bx is eliminated. This exploration should help them make the link before moving onto factorisation.

Highlight to students that, for example, in the expression $x^2 - 36$, the value of b is 0, hence why it is omitted. Then encourage students to consider a factor pair of –36 that sum to 0, to enable them to factorise the expression. Repeat with other expressions in this form, encouraging students to spot that the value of c is always a square number. If appropriate, explore fully factorising expressions such as $48 - 3y^2$

Misconceptions and common errors

- Students may not realise expressions in this form factorise into two brackets and they may try to factorise into one.
- Students may not use the correct signs when factorising, for example, factorising $x^2 - 49$ as $(x - 7)(x - 7)$.
- Students may think that expressions such as $3y^2 - 75$ are not a difference of two squares.

Mathematical talk

- What is meant by "the difference of two squares"?
- In a quadratic expression of the form $ax^2 \pm bx \pm c$, can the values of a, b or c be 0? Why or why not?
- What happens to the ax when you expand $(x - a)(x + a)$? Why?
- How do you know when an expression will factorise into one bracket or two?
- Why does the order of the brackets not matter when factorising?
- Explain why $x^2 + 121$ is not a difference of two squares.
- "x^2 – ______ factorises to $(x +$ ______$)(x -$ ______$)$"
  What numbers can you use to complete this statement?
  For example 16, 4 and 4

National Curriculum links

- Simplify and manipulate algebraic expressions (including those involving surds) by: factorising quadratic expressions of the form $x^2 + bx + c$, including the difference of two squares

F Difference of two squares E

Teaching approaches

- Begin by asking students to expand and simplify a quadratic expression.

  $$(x - 1)(x + 1) \equiv x^2 - x + x - 1 \equiv x^2 - 1$$

  Ask students what they notice about their answer.

  Highlight to students that there are only two terms as the x terms sum to zero.

  Repeat with similar expressions such as $(x + 5)(x - 5)$ before asking students to factorise expressions in the form $x^2 - a^2$

- Present students with a variety of expressions and ask them to identify which expressions are the difference of two squares.

$36 - x^2$	$r^2 - 10$	$y^2 - 49$	$t^2 + 10t + 25$
$81t - 9$	$w^2 + 36$	$2x^2 - 32$	$81t^2 - 9$

  Once students have correctly identified the expressions, model how to factorise. For example,

  $$36 - x^2 \equiv (6 + x)(6 - x)$$
  $$2x^2 - 32 \equiv 2(x^2 - 16) \equiv 2(x - 4)(x + 4)$$

Key vocabulary

variable symbol, usually a letter, that can represent any value in mathematical expressions, identities and formulae

expression collection of terms involving variables and numbers

factorise express an algebraic expression as a product of its factors using brackets; the reverse process of expanding brackets

quadratic equation or expression involving the second (and no higher) power of a variable or unknown, typically in the form $ax^2 + bx + c$

Links and next steps

- Main curriculum – Year 9 Spring Block 6 – Step 6 – Factorise quadratic expressions
- The difference of two squares can be used as a mental strategy for multiplication, for example, $19 \times 21 = (20 - 1)(20 + 1) = 20^2 - 1^2 = 399$

F

Solve quadratic equations equal to 0

White Rose MATHS

Notes and guidance

In this small step, students will solve quadratic expressions equal to zero in the form $(x + a)(x + b) = 0$. The purpose of this step is to prepare students for solving quadratic equations by factorisation. Firstly, practise solving linear equations equal to zero such as $x + 2 = 0$ and $ab = 0$, varying the value of a or b before solving.

Draw students' attention to the fact that if the product of two numbers or terms is zero, then at least one of the two numbers or terms must be zero. This understanding helps explain why quadratic equations can have up to two solutions. Remind students that $(x + a)(x + b) = 0$ represents $(x + a) \times (x + b) = 0$, and so to solve it, the value of either bracket must be 0

Mathematical talk

- x + ______ = 0
  What is the value of x?
- x – ______ = 0
  What is the value of x?
- If two numbers multiply to give 0, what do you know about one of the numbers?
- "Equations only have one solution."
  Give an example of where this true.
  Give an example of where this is not true.
- Why are there two solutions for x in equations such as $x(x + 1) = 0$ or $(x + 1)(x + 2) = 0$?
- How can you check if your solutions are correct?

Misconceptions and common errors

- Students may believe the solutions are the same as the constant within the brackets. For example, writing the solutions of $(x + 3)(x + 4)$ as $x = 3$ or $x = 4$

National Curriculum links

- Simplify and manipulate algebraic expressions (including those involving surds) by: factorising quadratic expressions of the form $x^2 + bx + c$, including the difference of two squares
- Solve quadratic equations algebraically by factorising; find approximate solutions using a graph

F Solve quadratic equations equal to 0

White Rose MATHS

Teaching approaches

- Display a set of equations and solutions and ask students to match together the correct pairs.

$2x = 0$	$x + 2 = 0$	$x - 2 = 0$
$x = -2$	$x = 0$	$x = 2$

- Display a factorised quadratic equation to students.

$$(x + 3)(x - 5) = 0$$

Model to students equating each bracket to zero and solving. Highlight to students that if one bracket is equal to zero, the whole expression will equal zero and hence there are two solutions.

$x + 3 = 0$ → -3 on both sides → $x = -3$

$x - 5 = 0$ → $+5$ on both sides → $x = 5$

Key vocabulary

solve find the value(s) or set of values of an unknown in an equation or inequality

solution value of an unknown that satisfies an equation or inequality

product result of a multiplication

Links and next steps

- Main curriculum – Year 9 Spring Block 6 – Step 7 – Solve quadratic equations (E)
- Students will later solve area problems that include quadratic equations.
- Challenge students to solve quadratic equations involving fractional solutions. For example, $(x + \frac{1}{2})(x - \frac{2}{3}) = 0$ or $(2x + 1)(3x - 2) = 0$

F Solve quadratic equations by factorisation

White Rose MATHS

Notes and guidance

In this small step, students will solve quadratic equations by first factorising the equation. Students may be familiar with solving quadratic equations from Year 9, as this was previously covered as an extend step. Building on students' understanding from the previous step, begin with factorising quadratic equations that are equal to zero before asking students to solve. Emphasise that quadratic equations must be equal to zero before they can be factorised.

Encourage students to check their solutions by substituting into the original equation. Once students are confident, challenge them to solve quadratic equations that are not equal to zero and require rearranging before solving, for example, $x^2 + 7x = -12$

Make links between solving a quadratic and the graphical representation of the quadratic using dynamic software.

Misconceptions and common errors

- Students may factorise the expression but not calculate the solutions when asked to solve.
- Students may not rearrange the expression if it is not equal to zero.
- Students may try to solve the equation by using a balanced method, similar to solving linear equations.

Mathematical talk

- What is the difference between factorising and solving?
- How can you factorise ______?
- What is meant by the solutions of a quadratic equation?
- How can you use your solutions to check your answer?
- Do all quadratic equations have two solutions? Explain how you know.
- "Quadratic equations must be equal to zero to be solved." Is the statement true or false? Explain how you know.
- How can $x^2 + 8x = -7$ be rearranged to equal zero?
- How is solving $x^2 + 4x = 0$ different to solving $x^2 + 4x + 3 = 0$?
- How is solving $x^2 + 9x = 0$ different to solving $x^2 - 9 = 0$?
- How do the solutions of the quadratic equation relate to the roots of the quadratic graph?

National Curriculum links

- Simplify and manipulate algebraic expressions (including those involving surds) by: factorising quadratic expressions of the form $x^2 + bx + c$, including the difference of two squares
- Solve quadratic equations algebraically by factorising; find approximate solutions using a graph

F

Solve quadratic equations by factorisation

White Rose MATHS

Teaching approaches

- Display a quadratic equation to students.

  Solve $x^2 - 8x + 12 = 0$

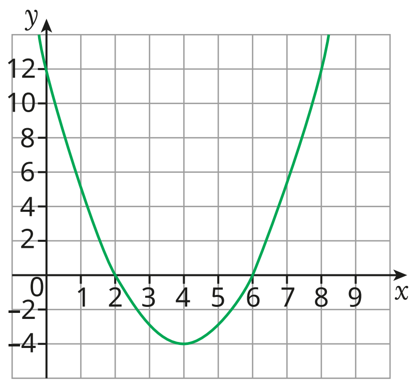
  Model to students factorising the equation before solving.

  $$(x - 2)(x - 6) = 0$$

  $x - 2 = 0$ or $x - 6 = 0$

  $x = 2$ $\quad$ $x = 6$

  Use dynamic graphing software to display the graph of $y = x^2 - 8x + 12$ to students.



  Ask students to discuss what they notice about the graph and the solutions for x.

  Repeat with other quadratic equations and ask students to discuss what they notice.

Key vocabulary

solution value of an unknown that satisfies an equation or inequality

factorise express an algebraic expression as a product of its factors using brackets; the reverse process of expanding brackets

roots value(s) of x where the quadratic equals 0

Links and next steps

- Main curriculum – Year 9 Spring Block 6 – Step 6 – Solve quadratic equations (E)
- Students will later solve area problems including quadratic equations.
- Challenge students to solve quadratic equations that are equal to each other, such as $2x^2 + 5x - 3 = x^2 - 5x - 24$

F

Quadratic graphs of the form $y = x^2 + a$

White Rose MATHS

Notes and guidance

In this small step, students will revisit previous content from Year 9 on plotting graphs in the form $y = x^2 \pm a$ using a table of values. Ensure students are confident substituting values, including negative numbers, into quadratic equations.

Draw students' attention to the fact quadratic graphs are drawn with a smooth curve and not straight lines.

Graphs in the form $y = x^2 \pm bx \pm c$ will be covered in later steps, so it is not necessary to address them now. If appropriate, challenge students to plot graphs with a negative coefficient of x^2, for example, $y = 5 - x^2$

Misconceptions and common errors

- Students may incorrectly square negative values. For example, writing $(-3)^2 = -9$
- Students may mistakenly join points up using straight lines instead of drawing a smooth curve.

Mathematical talk

- What is the value of ______ squared?
- What calculations need to be performed to calculate the value of the y-coordinate in $y = x^2 + 6$?
- What is the same and what is different about the table of values for the graphs $y = x^2$ and $y = x^2 + 3$?
- What is the same and what is different about the graphs $y = x^2$ and $y = x^2 + 3$?
- What does "draw the graph of $y = x^2 - 5$ for the values of x from –2 to 4" mean?
- Why do all quadratic graphs have a line of symmetry?
- What shape does the graph of a positive quadratic equation form?
- Explain the difference between the shape of a positive and negative quadratic graph.

National Curriculum links

- Recognise, sketch and produce graphs of linear and quadratic functions of one variable with appropriate scaling, using equations in x and y and the Cartesian plane (KS3)
- Recognise, sketch and interpret graphs of linear functions, quadratic functions, simple cubic functions, the reciprocal function $y = \frac{1}{x}$ with $x \neq 0$

F

Quadratic graphs of the form $y = x^2 + a$

Teaching approaches

- Show students two equations of quadratic graphs.

A $y = x^2$

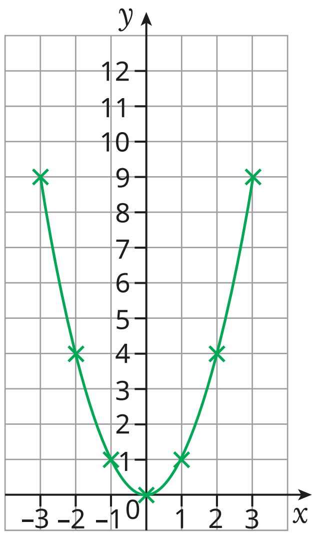
B $y = x^2 + 3$

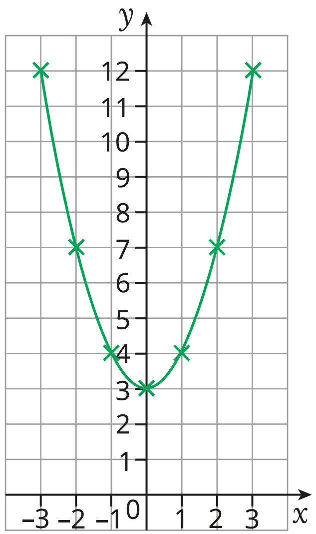
Ask students to discuss what they think will be the same and what will be different for the values of y.

Model to students how to complete a table of values before plotting on a coordinate grid.

x	–3	–2	–1	0	1	2	3
y	9	4	1	0	1	4	9

x	–3	–2	–1	0	1	2	3
y	12	7	4	3	4	7	12





Key vocabulary

quadratic equation or expression involving the second (and no higher) power of a variable or unknown, typically in the form $ax^2 + bx + c$

parabola approximately U-shaped symmetrical curve that often represents the graph of a quadratic function

table of values relationship between two variables (often x and y) organised by specific inputs (x) and outputs (y), which are often used to plot a graph or analyse the behaviour of a function

Links and next steps

- Main curriculum – Year 9 Summer Block 2 – Step 2 – Draw quadratic graphs
- Students will later solve quadratic equations graphically.
- Challenge students to calculate the table of values and plot the graphs of quadratics such as $y = 2x^2 - 5$

Autumn Block 4

Percentages

Small steps

Step 1	Percentage of an amount
Step 2	Percentage increase and decrease
Step 3	Repeated percentage change (E)
Step 4	Express one number as a fraction or percentage of another
Step 5	Express a change as a percentage
Step 6	Find the original value after a percentage change
Step 7	Simple interest
Step 8	Compound interest (E)
Step 9	Choose appropriate methods to solve percentage problems

F

Percentage of an amount

Notes and guidance

In this small step, students find percentages of an amount using both calculator and non-calculator methods. Students need to be confident with both and would benefit from having a bank of different methods for each to support them answering a variety of different question types. A good discussion point is around which method is most appropriate and efficient in each scenario.

Start by finding different percentages of simple numbers to focus on methods before increasing arithmetic complexity by using more challenging percentages and numbers. The use of multipliers supports efficiency by using a single calculation and supports students with later steps in the block. Students that followed the Support Curriculum in Year 9 may not be familiar with using multipliers to calculate percentages.

This step also provides an opportunity to incorporate contextual links, such as working out the amount of tax a person pays.

Misconceptions and common errors

- Students may think that percentages must be within 100%.
- Since calculating 10% means dividing by 10, students may assume that, for example, calculating 5% means dividing by 5
- Students may incorrectly convert a percentage to a decimal multiplier, for example writing 2% as 0.2

Mathematical talk

- What is a percentage?
- What is ______% as a decimal? How do you know?
- How do you find ______%?
- If you know ______%, how can you find ______%?
- How else can you find ______%?
- Which percentages can be worked out mentally?
- Do you need a calculator for this question? Why or why not?
- What does the % button on your calculator do?
- Will the answer always be greater or less than the original value? Explain how you know.

National Curriculum links

- Define percentage as 'number of parts per hundred', interpret percentages and percentage changes as a fraction or a decimal, interpret these multiplicatively, express one quantity as a percentage of another, compare two quantities using percentages, and work with percentages greater than 100% (KS3)
- Interpret fractions and percentages as operators (KS3)

Percentage of an amount

Teaching approaches

- Show students this information.

  100% is equivalent to 820

  Ask students questions to prompt their thinking.
  - What value is 50%/10%/25% equivalent to?
  - What other percentages can you work out?

  As a class, find different percentages of the same amount.

  Repeat using more challenging percentages and numbers, such as 12.5% of 98. Encourage students to use a calculator where appropriate.

- Ask students to write each percentage as a decimal.

  27% | 95% | 8.5%

  Ask them to use the decimals, and a calculator, to work out the percentages of the amounts.

  27% of 50 | 95% of 48 | 8.5% of 140

Key vocabulary

percentage expressing a part of a whole as a number out of 100

decimal number number with a whole and fractional part, for example 3.2, 5.43

multiplier number that is multiplied with another number to increase or decrease its value

whole total or complete amount

Links and next steps

Knowledge of percentages is required in various areas of science, such as finding the percentage composition in certain compounds.

- Support curriculum – Year 9 Autumn Block 2 – Step 2 – Percentage of an amount
- Main curriculum – Year 8 – Spring Block 3 – Step 1 – Percentage of an amount
- Main curriculum – Year 8 – Spring Block 3 – Step 3 – Use multipliers to calculate percentages

F

Percentage increase and decrease

White Rose MATHS

Notes and guidance

Building on from the previous small step, students use calculator and non-calculator methods to increase and decrease by a given percentage. If students followed the Support Curriculum in Year 9, they may have only increased or decreased with percentages that are multiples of 5%.

When answering these questions, a useful starting question is to ask students, "Will the answer be greater than or less than the original amount?", linking to their existing understanding of the words "increase" and "decrease". This will support them in choosing the appropriate steps to complete their calculations, as well as being able to sense check their answers. Where the question doesn't explicitly state the word "increase" or "decrease", prompt students to interpret the context and recognise what is being asked.

Misconceptions and common errors

- Students may mix up percentage increase and decrease.
- Students may calculate the percentage of an amount correctly but forget to add or subtract this from 100%.
- Students may use multipliers incorrectly, for example to increase by 2.5% they may multiply by 1.25 instead of 1.025

Mathematical talk

- Will the answer be greater than or less than this amount? How do you know?
- Is this an increase or a decrease? How do you know?
- What is ______% as a decimal?
- Can you work it out another way?
- Do you need a calculator for this question? Explain how you know.
- Is 0.65 the correct multiplier to decrease an amount by 65%? Explain how you know.
- Can a number increase by more than 100%? Why/why not?

National Curriculum links

- Define percentage as 'number of parts per hundred', interpret percentages and percentage changes as a fraction or a decimal, interpret these multiplicatively, express one quantity as a percentage of another, compare two quantities using percentages, and work with percentages greater than 100% (KS3)
- Solve problems involving percentage change, including: percentage increase, decrease and original value problems and simple interest in financial mathematics (KS3)
- Interpret fractions and percentages as operators (KS3)

F Percentage increase and decrease

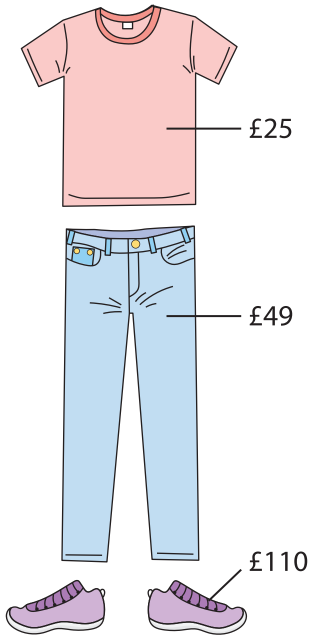
Teaching approaches

- Show students the price of some items of clothing.

  Tell them that there is a 30% sale on.

  Ask students questions to develop their understanding.

  - Will the price of each item go up or down in the sale? Why?
  - Is this a percentage increase or decrease?
  - How can you calculate the sale price of each item?



- Tell students that the average UK house price increased by 73% between 2013 and 2023

  Ask students questions to prompt their thinking.

  - Were houses more expensive in 2013 or in 2023?
  - If the value of a house was £125 000 in 2013, how can you work out its value in 2023?

Key vocabulary

percentage	expressing a part of a whole as a number out of 100
multiplier	number that is multiplied with another number to increase or decrease its value
increase	make a number greater
decrease	make a number smaller

Links and next steps

Students will apply percentage increase and decrease in various scientific scenarios. For example, they might calculate the final mass of an object, given its initial mass and a percentage increase or decrease.

- Support curriculum – Year 9 Autumn Block 2 – Step 3 – Percentage increase and decrease
- Main curriculum – Year 9 Autumn Block 2 – Step 1 – Percentage increase and decrease
- Challenge students to apply successive percentage increases/decreases to a starting amount.

F Repeated percentage change E

White Rose MATHS

Notes and guidance

In this small step, students explore repeated percentage change involving both percentage increases and decreases. This will support students when later exploring compound interest, which is a specific example of repeated percentage change.

Examples should include those where the percentage an amount is increasing or decreasing by remains constant, and also examples where it changes. Students should also explore growth and decay problems.

Multipliers are the most efficient method when the repeated change is with the same percentage. Encourage discussions around why the sum of individual percentage changes does not equate to their combined effect over time. For example, a 5% increase followed by a 10% increase is not equivalent to a 15% increase of the original value.

Misconceptions and common errors

- Students may find the percentage of the original amount rather than recalculating it for each time period.
- Students may lose track of the number of times they have completed the percentage change.
- Students may think that decreasing by 10% and then 20% is equivalent to decreasing by 30%.

Mathematical talk

- Will the answer be greater than or less than this amount? How do you know?
- Is this an increase or a decrease? How do you know?
- Do you always find the percentage of the same amount? Why or why not?
- Is increasing by 10% then 20% the same as increasing by 30%? Explain how you know.
- Explain why increasing by 10% and then decreasing by 10% does not return to the original amount.
- How many times will a percentage increase of 10% need to be repeated until the value is double the original?

National Curriculum links

- Interpret fractions and percentages as operators (KS3)
- Solve problems involving percentage change, including: percentage increase, decrease and original value problems and simple interest in financial mathematics (KS3)
- Set up, solve and interpret the answers in growth and decay problems, including compound interest

F Repeated percentage change E

White Rose MATHS

Teaching approaches

- Show students this information.

> The value of a new car depreciates by:
>
> 20% during the first year
>
> 30% during the second year
>
> 40% during the third year

The value of a car when new is £45 000

Ask students questions to develop their understanding.

- What was the value after 1 year/2 years/3 years?
- Do you always find the percentage of £45 000?
- Has the value depreciated by 90% overall after 3 years?

- Tell students a population of bacteria increases by 18% every 2 hours and the population starts at 12 000

Ask students questions to check their understanding.

- What is the population after 2 hours/8 hours?
- What else can you work out?

Key vocabulary

repeated percentage change	applying a percentage increase or decrease multiple times in succession to an amount
multiplier	number that is multiplied with another number to increase or decrease its value
depreciate	decrease in value over time
index	written as a small number to the right and above the base number, indicating how many times the base has been multiplied by itself

Links and next steps

- Knowledge of repeated percentage change is required in various areas of science such as radioactive decay or bacterial growth.
- Main curriculum – Year 9 – Autumn Block 2 – Step 6 – Repeated percentage change
- Challenge students to use roots of higher powers to determine how many times the percentage change needs to be repeated to reach a certain value.

F

Express one number as a fraction or percentage of another

White Rose MATHS

Notes and guidance

In this small step, students express one number as a fraction or a percentage of another using both calculator and non-calculator methods. It is useful to start with fractions, as students' common terminology around fractions will support their understanding. For example, $\frac{11}{50}$ is often read as "eleven fiftieths" or "11 out of 50", and the latter supports this learning. Once confident in this, students can then use their understanding of percentages being out of 100 and equivalent fractions to express one number as a percentage of another, before moving on to more complex examples where they express any number as a fraction or percentage of another. Encourage students to check their answers by calculating the fraction or percentage of the amount to ensure it is accurate.

Misconceptions and common errors

- If decimals are involved in the question, students may not write their fraction with integers. For example, if expressing 2.3 as a fraction of 4, students may give their answer as $\frac{2.3}{4}$
- Students may mix up the order of the numbers in the question and hence get an incorrect answer.
- Students may not understand recurring decimal notation.

Mathematical talk

- How do you say this fraction?
  How else can you say it?
- What is the equivalent fraction to this that has 100 as the denominator?
- What is this fraction as a percentage?
- Without using equivalent fractions, how else can you convert the fraction to a percentage?
- How can you check your answer?
- How can you type a fraction on your calculator?
- Why might you need a calculator to calculate the percentage of a test mark out of 35, but not for a test mark out of 25?

National Curriculum links

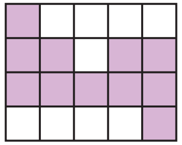
- Define percentage as 'number of parts per hundred', interpret percentages and percentage changes as a fraction or a decimal, interpret these multiplicatively, express one quantity as a percentage of another, compare two quantities using percentages, and work with percentages greater than 100% (KS3)
- Express one quantity as a fraction of another, where the fraction is less than 1 and greater than 1 (KS3)

F Express one number as a fraction or percentage of another

White Rose MATHS

Teaching approaches

- Show students this diagram.



Ask students questions to prompt their thinking.

  - How many squares are there in total?
  - How many of the squares are shaded?
  - What fraction of the squares are shaded?
  - How can you rewrite this fraction with a denominator of 100?
  - What percentage of the squares are shaded?

- Present students with a problem.

> A test is out of 40 marks.
> Ron scores 24 marks.
> What is his mark as a percentage?

As a class, explore a range of methods to solve the problem and encourage discussion on how to check the solution by using the percentage found to calculate that percentage of 40

Key vocabulary

express rewrite in another way after some working

fraction number with a numerator and a denominator that represents a proportion

percentage expressing a part of a whole as a number out of 100

equivalent fraction fraction that represents the same value as another fraction with different numerators and denominators

Links and next steps

- Knowledge of expressing one number as a fraction or percentage of another is required in various areas of science, for example when calculating energy efficiency.
- Support curriculum – Year 9 Autumn Block 2 – Step 5 – Express one number as a percentage of another
- Main curriculum – Year 8 – Spring Block 3 – Step 9 – Express one number as a fraction or percentage of another (calculator)

F

Express a change as a percentage

Notes and guidance

Building on from the previous small step, students now write a change between two amounts as a percentage. Encourage students to identify and label the original and new amounts so that they can correctly identify whether the change is an increase or decrease. From this, they then calculate the difference in the amounts and use the skills from the previous step to express this difference as a percentage of the original amount.

Students become familiar with the general formula $\frac{\text{change}}{\text{original}} \times 100$ to express the percentage change.

It is useful to apply contexts in this step, such as changes in population figures. This skill also commonly arises in science questions. For example, when investigating osmosis, students calculate the percentage change in the mass of a potato.

Misconceptions and common errors

- Students may confuse whether the percentage change is an increase or decrease if they mix up the numbers given in the question.
- Students may divide the difference by the new amount instead of the original amount.
- Students may express one amount as a percentage of another, rather than expressing the change as a percentage.

Mathematical talk

- What was the original amount?
- What is the new amount?
- Has the original amount increased or decreased?
- What has it increased/decreased by?
- What is the difference between the given values?
- How can you express the difference as a fraction of the original amount?
- How can you convert the fraction into a percentage?
- What degree of accuracy is appropriate for expressing the percentage change?
- When might you use percentage change in everyday life?

National Curriculum links

- Define percentage as "number of parts per hundred", interpret percentages and percentage changes as a fraction or a decimal, interpret these multiplicatively, express one quantity as a percentage of another, compare two quantities using percentages, and work with percentages greater than 100% (KS3)
- Solve problems involving percentage change, including: percentage increase, decrease and original value problems and simple interest in financial mathematics (KS3)

F Express a change as a percentage

White Rose MATHS

Teaching approaches

- Show students this information.

Number of subscribers	
2023	180 000
2024	211 000

Ask students questions about the information.

- What was the number of subscribers originally?
- What did it change to?
- Did the number of subscribers increase or decrease?
- How much did the number of subscribers increase/decrease by?
- How can you write this change as a fraction of the original value?
- How can you convert this fraction to a percentage?
- What is the percentage change in subscribers from 2023 to 2024?

Emphasise that this is known as the "percentage change".

Repeat with other problems with different contexts.

Key vocabulary

percentage expressing a part of a whole as a number out of 100

percentage change how much a quantity has increased or decreased relative to its original value, calculated as a percentage

profit positive difference between the selling price and the cost price of an item, indicating financial gain

loss negative difference between the cost price and the selling price of an item, indicating a financial loss

Links and next steps

- Students will apply percentage change in various scientific contexts. For example, they might find the percentage change in the mass of a potato when exploring osmosis.
- Main curriculum – Year 9 – Autumn Block 2 – Step 2 – Express a change as a percentage
- Challenge students to calculate the percentage change of values in standard form. For example, the change in the population of a city over time.

F

Find the original value after a percentage change

Notes and guidance

Building on the previous small step, students now find the original value given a percentage or a percentage change. Ensure that students understand that the original amount is represented by 100%. Using and comparing common everyday life contexts can support students in understanding the difference between this step and the earlier step on calculating a percentage of an amount. Prompt students to consider whether the given amount represents 100% and explain why.

Bar models can also be a useful tool to support students in visualising information and highlighting the percentage that the amount represents. They can be particularly useful when students are faced with a mixture of questions and deciding which skill is being tested.

Misconceptions and common errors

- Students may just calculate the percentage of the given amount, instead of finding the original amount.
- Students may find the percentage of the given amount, then add/subtract it to/from the amount, rather than using the percentage to work out the original amount.
- Students may misinterpret the context of the problem and think that the problem represents a percentage increase when it is describing a decrease.

Mathematical talk

- Is this value 100%? Explain how you know.
- Has there been a percentage increase or decrease? How do you know?
- Was the original number greater or less than this? How do you know?
- What percentage does this number represent?
- If you know ______%, how can you find ______%?
- Should your answer for the original value be greater or less than the given value? Explain how you know.
- How can you check your answer?

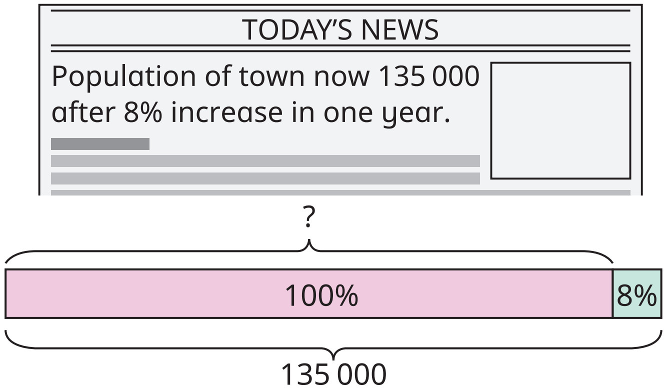
National Curriculum links

- Define percentage as 'number of parts per hundred', interpret percentages and percentage changes as a fraction or a decimal, interpret these multiplicatively, express one quantity as a percentage of another, compare two quantities using percentages, and work with percentages greater than 100% (KS3)
- Solve problems involving percentage change, including: percentage increase, decrease and original value problems and simple interest in financial mathematics (KS3)

F Find the original value after a percentage change

Teaching approaches

- Show students some information and supporting bar model.



Ask students questions to prompt their thinking.

- What percentage does 135 000 represent?
- What percentage does the population last year represent?
- Does decreasing 135 000 by 8% find the population last year?
- If you know 108%, how can you work out 1%?
- If you know 1%, how can you work out the population last year?
- What was the population of the town last year?

Encourage students to verify their answer by increasing it by 8% to check it gives them 135 000

Key vocabulary

original	initial value or quantity before any changes
change	difference between an initial and final value
increase	make a number greater
decrease	make a number smaller
depreciate	decrease in value over time

Links and next steps

- Students will find the original value in a variety of scientific contexts, for example when considering changes in animal populations.
- Support curriculum – Year 9 Autumn Block 2 – Step 4 – Find the original value given a percentage
- Main curriculum – Year 9 Autumn Block 2 – Step 3 – Find the original value after a percentage change
- Challenge students to determine the pre-tax income from a given after-tax income.

F

Simple interest

White Rose MATHS

Notes and guidance

In this small step, students explore how to calculate simple interest. They need to understand that simple interest means that the amount of interest added does not change, and they do not need to recalculate the interest at the end of each time period.

This step is a great opportunity to include elements of financial literacy to prepare them for life beyond education. Whilst not the most common type of interest in real life, simple interest can be applied in products such as personal loans, short-term investments and bonds.

Whilst compound interest isn't covered until the next step, it can be useful to discuss the benefits of simple interest with students by asking questions such as, "Is it positive or negative that the interest rate is always calculated on the original amount? Why?".

Misconceptions and common errors

- Students may incorrectly interpret the question, for example working out the total interest instead of the total value of the investment, or vice versa.
- As students have covered both simple and compound interest earlier in their education, they may confuse the two and apply a compound interest method.
- Students may not be familiar with certain time period units such as "per annum".

Mathematical talk

- What is interest?
- What is simple interest?
- Are there any other types of interest?
- What is the interest rate given in the question?
- How often is the interest charged/paid?
- How can you work out the total amount of interest?
- How does the interest rate affect the total amount earned or owed?
- Is simple interest that is earned annually always the same value each year? Explain how you know.

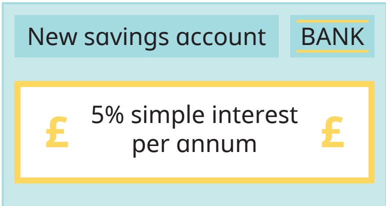
National Curriculum links

- Define percentage as 'number of parts per hundred', interpret percentages and percentage changes as a fraction or a decimal, interpret these multiplicatively, express one quantity as a percentage of another, compare two quantities using percentages, and work with percentages greater than 100% (KS3)
- Solve problems involving percentage change, including: percentage increase, decrease and original value problems and simple interest in financial mathematics (KS3)
- Interpret fractions and percentages as operators (KS3)

F Simple interest

Teaching approaches

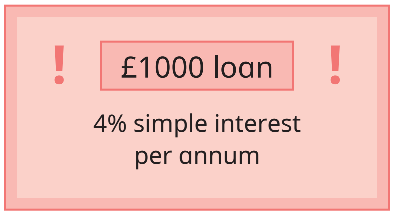
- Show students this information.



Ask students questions to develop their understanding.

  - What does simple interest mean?
  - What does per annum mean?
  - If £250 is deposited, how can you work out how much interest is earned after 1 year?
  - If £250 is deposited, how can you work out how much money is in the account after 5 years?

- Show some more information.



Ask students to calculate the total interest owed if the loan is repaid over 5 years. Then, have them determine the monthly repayment amounts over the same period.

Key vocabulary

simple interest	interest calculated on the principal amount and then multiplied by the time period of the investment or loan
deposit	money placed in a bank account or given as an initial payment for a service or purchase
investment	amount deposited in a savings account with the goal of earning a financial gain or increasing the value over time
per annum	"per year", used to describe interest rates or payments that occur annually

Links and next steps

- Support curriculum – Year 9 Spring Block 2 – Step 7 – Borrowing (loans)
- Main curriculum – Year 9 Autumn Block 2 – Step 8 – Simple interest
- Challenge students to solve simple interest problems using time periods that are not in years.

F Compound interest E

Notes and guidance

In this small step, students explore compound interest, understanding how it differs from simple interest. Students understand that, unlike simple interest, the amount of interest changes each time period based on the new value. This small step is an application of repeated percentage change.

To further support financial literacy, consider spending time comparing and contrasting simple and compound interest and the practical implications of each. When teaching compound interest, it can be useful to work through and set out each time period individually to clearly highlight the differences between compound and simple interest, before moving on to a multiplier method for efficiency.

Misconceptions and common errors

- Students may apply a simple interest method to a compound interest question.
- Students may use the multiplier to find the percentage of an amount, rather than the multiplier to increase/decrease by the percentage.
- Students may misread the question and give the total amount when just asked for the interest, or vice versa.

Mathematical talk

- What is interest?
- What is compound interest?
- What is the same and what is different about simple interest and compound interest?
- What is the interest rate given in the question?
- How often is the interest charged/paid?
- Why is the amount of interest earned different every year?
- How can you work out the total amount of interest?
- How can you work it out using multipliers?
- Why does compound interest grow faster than simple interest over time?
- Why do banks and lenders use compound interest instead of simple interest?

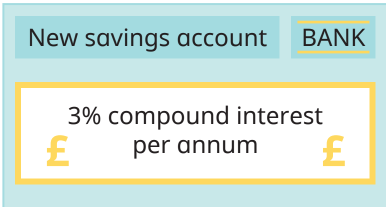
National Curriculum links

- Interpret fractions and percentages as operators (KS3)
- Set up, solve and interpret the answers in growth and decay problems, including compound interest

F Compound interest E

Teaching approaches

- Show students this information.



Ask students questions to develop their understanding.

- What does compound interest mean?
- What does per annum mean?
- If £250 is invested, how can you work out how much money is in the account after 5 years?

Model step-by-step working before showing them a multiplier method.

$103\% = 1.03$

$$\underbrace{£250}_{\text{original investment}} \times \underbrace{1.03 \times 1.03 \times 1.03 \times 1.03 \times 1.03}_{\text{5 times for 5 years}} = £250 \times (1.03)^5$$

Consider working out the problem using simple interest and comparing the difference between the two results.

Key vocabulary

compound interest	interest calculated on both the principal amount and the accumulated interest from previous periods
interest rate	percentage used to calculate interest on an investment or a loan
multiplier	number that is multiplied with another number to increase or decrease its value
per annum	"per year", used to describe interest rates or payments that occur annually

Links and next steps

- Students will apply their understanding of compound interest to calculations involving growth and decay, such as determining half-life.
- Main curriculum – Year 9 Autumn Block 2 – Step 9 – Compound interest
- Challenge students to solve compound interest problems using time periods that are not in years.

F

Choose appropriate methods to solve percentage problems

White Rose MATHS

Notes and guidance

This small step allows students the opportunity to practise a diverse range of percentage problems where their choice of method is not prescriptive but may impact on efficiency and accuracy. Both calculator and non-calculator problems should be explored. Ensure there are questions where the original amount is given, or to be found from a percentage, to tackle students' overgeneralisations such as always finding the percentage of the given amount first.

Bar models may support students in choosing the correct calculation to solve the problem. If appropriate, students could explore using properties of commutativity to solve percentage problems, for example, 64% of 25 = 25% of 64

Encourage students to check whether their answer is reasonable by considering it in the context of the question.

Misconceptions and common errors

- Students may use inefficient methods on their calculator. For example, finding 30% by using 10% + 10% + 10% instead of × 0.3
- Students may calculate the percentage of the given amount without considering whether this amount is the whole or not.

Mathematical talk

- How do you find ______% of a number? For example, 30%. Tell me a different way. How many different ways can you find?
- Explain the sequence of buttons you can press on your calculator to show that 25% of 18 = 4.5
- Give an example of a percentage that can be found without using a multiplier.
- "It is always easier to use a multiplier to find a percentage." Give an example of where this may not be true.
- Should your answer be less than or greater than the original value? Explain how you know.
- How can you check your answer?

National Curriculum links

- Define percentage as 'number of parts per hundred', interpret percentages and percentage changes as a fraction or a decimal, interpret these multiplicatively, express one quantity as a percentage of another, compare two quantities using percentages, and work with percentages greater than 100% (KS3)
- Solve problems involving percentage change, including: percentage increase, decrease and original value problems and simple interest in financial mathematics (KS3)

F Choose appropriate methods to solve percentage problems

Teaching approaches

- Show students a problem with model answers.

A jumper costs £25.
The price increases by 6%.
Work out the new price of the jumper.

Method 1	Method 2
10% of £25 = £2.50	100% + 6% = 106%
5% of £25 = £1.25	106% = 1.06
1% of £25 = £0.25	£25 × 1.06 = £26.50
6% of £25 = £1.50	
new price = £26.50	

Ask students to compare the methods, encouraging them to consider which is suitable with/without a calculator.

Repeat with other prices such as £42.25, encouraging the same discussions.

- Use procedural variation to support students in focusing on what is required from the problem.

If 30% of a number is 64, what is the number?

A bakery sold 30 cakes on Monday and 64 cakes on Tuesday. Work out the percentage change.

Decrease 64 by 30% and then again by 30%.

Find 30% of 64

Write 30 out of 64 as a percentage.

Key vocabulary

percentage change how much a quantity has increased or decreased relative to its original value, calculated as a percentage

multiplier number that is multiplied with another number to increase or decrease its value

commutative property where an operation gives the same result whatever the order of the terms involved

Links and next steps

- Knowledge of percentages is required in various areas of science, such as osmosis, percentage composition in compounds and bacterial growth.
- Support curriculum – Year 9 Autumn Block 2 – Step 6 – Solve problems with percentages
- Main curriculum – Year 9 Autumn Block 2 – Step 5 – Solve problems with percentages (calculator)
- Solving problems with percentages can be interleaved into many other areas of the curriculum at Key Stage 4, such as ratio and interpreting data.