Autumn
Scheme of learning
Year 11

#MathsEveryoneCan

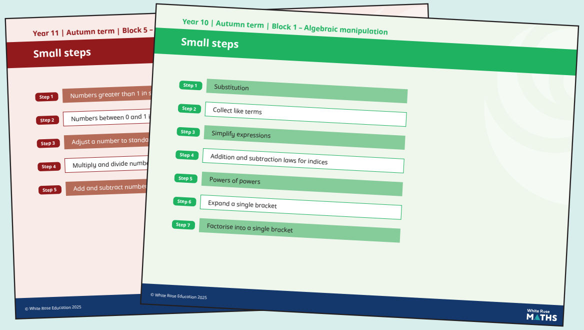
White Rose MATHS

The **White Rose Maths** schemes of learning

Why small steps?

We know that if too many concepts are covered at once, it can lead to cognitive overload, so we believe it is better to follow a small steps approach to the curriculum. As a result, each block of content in our schemes of learning is broken down into small manageable steps.

It is not the intention that each small step should last a lesson – some will be a short step withing a lesson; some will take longer than a lesson. We encourage teachers to spend the appropriate amount of time on each step for their group, and to teach some steps alongside each other if necessary.



Teaching for mastery

Our research-based schemes of learning are designed to support a mastery approach to teaching and learning and are consistent with the aims and objectives of the National Curriculum.

Curriculum structure and sequencing

Key concepts are introduced in a carefully considered logical sequence, allowing students to build a deep and connected understanding. This consistent and coherent approach supports students in making links within and between topics over time, laying secure foundations for future learning.

Evidence-informed design

Our curriculum is designed by a team of maths specialists, with an emphasis on how students learn, retain and retrieve knowledge. We incorporate strategies such as retrieval practice and spaced learning to support long-term memory and deepen understanding. The use of models and representations plays a key role in reducing cognitive load and making abstract concepts more accessible. Informed by the latest educational research, each scheme of learning also identifies common misconceptions and provides targeted approaches to address them, supporting effective, evidence-informed teaching in every classroom.

Fluency, reasoning and problem solving

Our schemes develop all three key areas of the National Curriculum, giving students the knowledge and skills they need to become confident mathematicians.

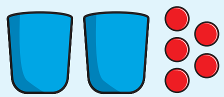
The **White Rose Maths** schemes of learning

Concrete – Pictorial – Abstract (CPA)

Research shows that all students, when introduced to a new concept, should have the opportunity to build competency by following the CPA approach. This features throughout our schemes of learning.

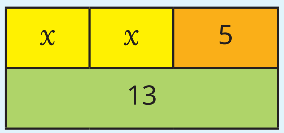
Concrete

Students should have the opportunity to work with physical objects/concrete resources, in order to bring the maths to life and to build understanding of what they are doing.



Pictorial

Alongside concrete resources, students should work with pictorial representations, making links to the concrete. Visualising a problem in this way can help students to reason and to solve problems.



Abstract

With the support of both the concrete and pictorial representations, students can develop their understanding of abstract methods.

$2x + 5 = 13$

Key Stage 3 and 4 symbols

The following symbols are used to indicate:

- concrete resources might be useful to help answer the question
- a bar model might be useful to help answer the question
- drawing a picture might help students to answer the question
- students talk about and compare their answers and reasoning
- a question that should really make students think. The question may be structured differently or require a different approach from others and/or tease out common misconceptions.
- the step has an explicit link to science, helping students to make cross-curricular connections.

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Teacher guidance

Every block in our schemes of learning is broken down into manageable small steps, with comprehensive teacher guidance for each one. Here are the features included in each step.

Notes and guidance provide an overview of the content of the step, and ideas for teaching, along with advice on progression and where a topic fits within the curriculum.

Misconceptions and common errors are highlighted, as well as areas that may require additional support.

Year 10 | Autumn term | Block 1 – Algebraic manipulation | Step 1

F **Substitution**

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Notes and guidance

In this small step, students consolidate their understanding and application of substituting given values into algebraic expressions and formulae.

It is important for students to practise substituting values into a wide variety of expressions and formulae, including those with coefficients, exponents, multiple terms, and fractions. They should also work with a mixture of values, such as integers, negative numbers, decimals and fractions.

Ensure that students practise substitution with and without a calculator. When using a calculator, highlight that, for example evaluating $3a^2$ when $a = 4$, can be calculated by typing 3×4^2 or $3(4)^2$

Mathematical talk

- What is meant by substitution?
- What does evaluate mean?
- What order should the operations be completed in?
- What is the same and what is different about each pair of expressions: $2a$ and a^2, $\frac{a}{b}$ and $\frac{b}{a}$, and $a - b$ and $b - a$?
- "The greater the substituted value, the greater the value of the expression." Give an example where this is not true.
- "If x is negative then the value of an expression containing x will be negative." Explain why this is not always true.
- If b is negative, explain why $10 - 2b$ will always be positive.
- What value of g cannot be substituted into $\frac{g}{(g-3)}$? Explain your answer.

Misconceptions and common errors

- Students may incorrectly interpret expressions. For example, thinking $3a^2$ is equivalent to $(3a)^2$
- Students may incorrectly substitute negative numbers, especially when there is a negative coefficient of the variable. For example, substituting $a = -3$ into $7 - 2a$.

National Curriculum links

- Substitute numerical values into formulae and expressions, including scientific formulae (KS3)

Mathematical talk provides key questions, discussion points and possible sentence stems that can be used to develop students' mathematical vocabulary and reasoning skills, digging deeper into the content.

National Curriculum links to indicate the objective(s) being addressed by the step.

Teacher guidance

Teaching approaches offer practical strategies for classroom use, including effective representations, modelled examples and key questions or activities designed to promote reasoning and problem solving.

Year 10 | Autumn term | Block 1 – Algebraic manipulation | Step 1

F Substitution

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Teaching approaches

- Display cards showing expressions and values.

$\frac{b}{c} - a$ | $b - \frac{a}{c}$ | $\frac{b - a}{c}$ | $a = 0.5$ | $b = -2$ | $c = 4$

Ask students questions to support them with the substitution.

- Which expressions require a division before the subtraction?
- Which expressions will be positive? Which will be negative?

Repeat with different expressions and values.

- Show students a formula.

A rugby union team scores T points in a match.

$T = 7c + 5u + 3p$

c = number of converted tries scored
u = number of unconverted tries scored
p = number of penalty kicks or drop goals scored

Ask students questions about the formula.

- What is the value of T if $c = 2$, $u = 1$, $p = 5$?
- If $T = 15$, what could c, u and p be?

Key vocabulary

expression	collection of terms involving variables (letters) and numbers
variable	symbol, usually a letter, that can represent any value in mathematical expressions, identities and formulae
formula	mathematical rule or equation that shows the relationship between different variables or quantities
substitute	replace letters with numerical values

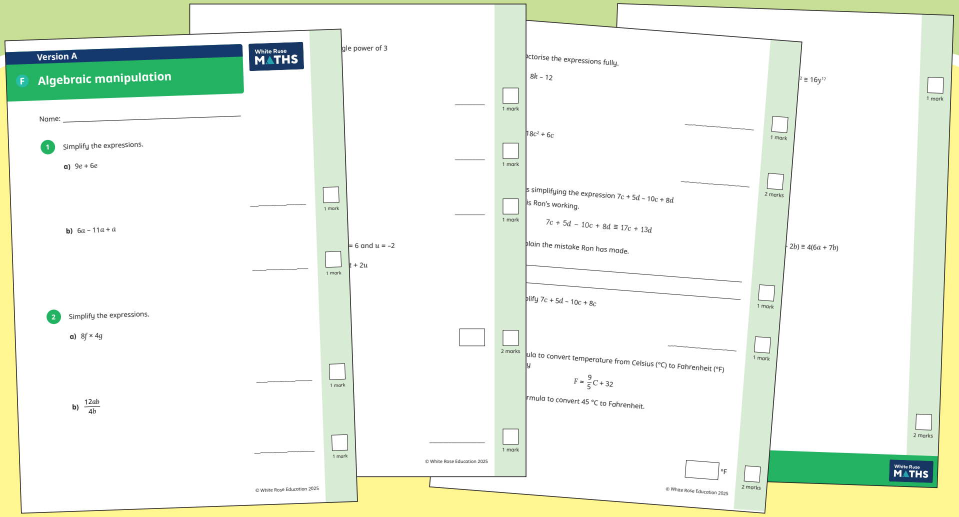
Links and next steps

- Students will substitute into a variety of scientific formulae, such as Newton's Second Law.
- Support Curriculum – Year 8 Spring Block 5 – Step 4 – Substitution
- Main Curriculum – Year 9 Autumn Block 4 – Step 6 – Substitute into formulae and equations
- Being fluent in substitution is an essential technique and is a part of the process of solving a pair of linear simultaneous equations.

Key vocabulary emphasises the importance of mathematical language and offers clear, age-appropriate definitions to support understanding

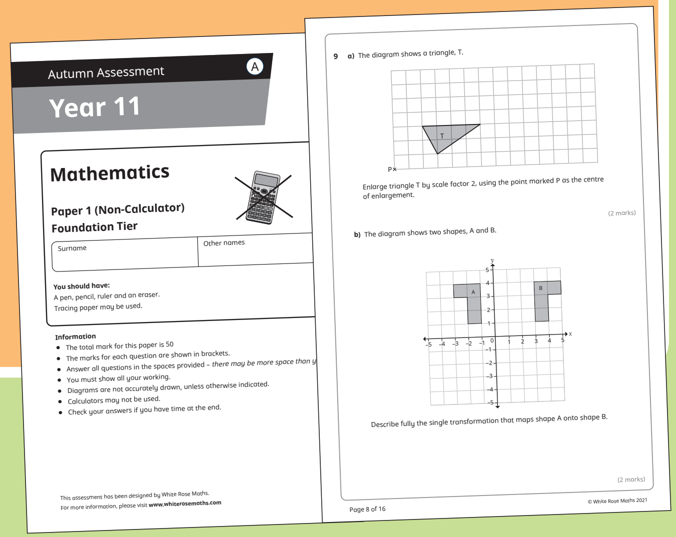
Links and next steps highlight connections to science (where appropriate) as well as alignment with the Support curriculum and shows how this step builds towards future learning. It may also include a challenge to deepen understanding, while remaining within the scope of the small step.

Free supporting materials



End-of-block assessments are provided for teachers to see how well students are progressing with the material in the curriculum. These have a total of 20 marks, assessing students' understanding of all of the steps within a block. These can be used flexibly – in the classroom, as homework, with/without a calculator, immediately after a block or later in the year – to suit teachers' and students' needs. Answers are provided.

End-of-term assessments are also provided for teachers to assess how well material is being learnt and retained in the medium and long term. There will be a calculator and non-calculator paper provided for the end of each term for each Years 7, 8 and 9. All papers will have a total of 40 marks available. We suggest 45 minutes for a paper, so that they can be done within a typical lesson. Mark schemes are provided.



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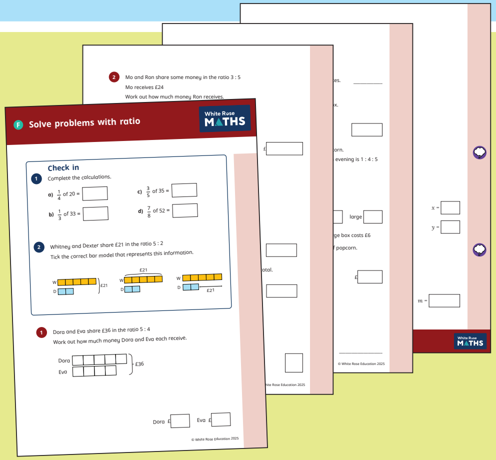
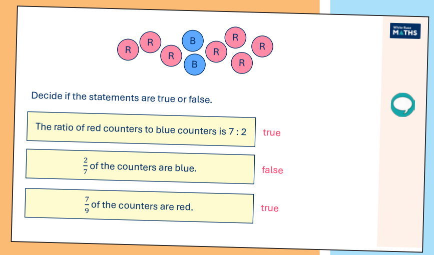
Premium supporting materials

Worksheets to accompany every small step, providing relevant practice questions for each topic that will reinforce learning at every stage.

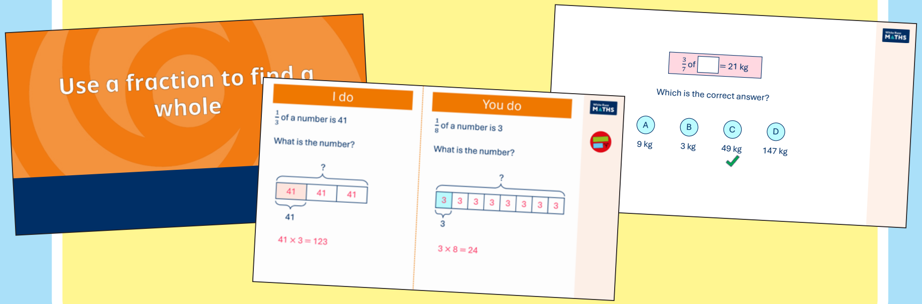
Answers to all the worksheet questions are provided.



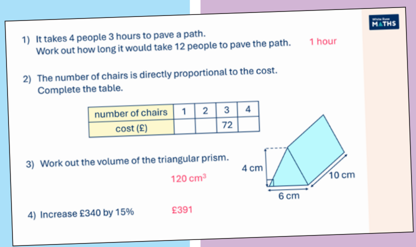
A **true or false** question for every small step in the scheme of learning. These can be used to support new learning or as another tool for revisiting knowledge at a later date.



Teaching slides for every small step, providing worked examples, multiple choice questions and open-ended questions. These are fully animated and editable, so can be adapted to the needs of any class.



Flashback 4 starter activities to improve retention.
Q1 is from the last lesson;
Q2 is from last week;
Q3 is from 2 to 3 weeks ago;
Q4 is from last term/year.



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Yearly overview

The yearly overview provides suggested timings for each block of learning, which can be adapted to suit different term dates or other requirements.

	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10	Week 11	Week 12
Autumn	Ratio, proportion and rates of change **Ratio and proportion**		Geometry and measures **Area and volume**		Geometry and measures **Similarity and congruence**		Algebra **Sequences and proof**		Number **Standard form**	Geometry and measures **Work with circles**		Probability **Set notation and Venn diagrams**
Spring	Algebra **Functions and graphs**		Algebra **Equations and formulae**		Ratio, proportion and rates of change **Rates**		Geometry and measures **Angles, bearings and trigonometry**		Geometry and measures **Constructions and loci**		Geometry and measures **Transformations**	
Summer	Revision and exams											

Autumn Block 1

Ratio and proportion

Small steps

Step 1	Solve problems with ratio
Step 2	Direct proportion
Step 3	Best buy problems
Step 4	Conversion graphs
Step 5	Exchange rates
Step 6	Inverse proportion
Step 7	Direct proportion equations (E)
Step 8	Direct and inverse proportion graphs (E)

F

Solve problems with ratio

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Notes and guidance

In this small step, students will revisit problems involving ratio covered in Year 10. Provide opportunities for students to share amounts in a ratio when given either a total, a part, or a difference.

Bar models can help students to identify key differences between problems and to establish a suitable strategy to solve them, rather than relying on a procedural method, such as dividing an amount by the sum of the parts in a given ratio.

Students should also explore links between fractions and ratios. Note that "combine a set of ratios" and "link ratio and algebra" are both Extend steps in Year 10 and may not have been covered previously. Time could be given to each of these steps if appropriate.

Misconceptions and common errors

- Students may misinterpret a ratio problem, for example, interpreting a part as a total.
- Students may misinterpret links between fractions and ratios, for example, if the number of red counters to the number of blue counters is 2 : 3 then they may state that $\frac{2}{3}$ of the counters are red.
- Students may not draw bar models accurately, leading to incorrect calculations.

Mathematical talk

- Is 2 : 3 the same as 3 : 2? Explain how you know.
- Which person receives more? How do you know?
- How many parts does ______ represent?
- How many parts are there in total?
- Does the amount given represent a total or a part? How do you know?
- Which calculations are represented by the bar model?
- "To share in a ratio, add the parts of the ratio together then divide the amount by the total number of parts." Give an example of where this is true. Give an example of where this is false.
- How do you express a ratio as a fraction?

National Curriculum links

- Use ratio notation, including reduction to simplest form (KS3)
- Divide a given quantity into two parts in a given part : part or part : whole ratio; express the division of a quantity into two parts as a ratio (KS3)
- Understand that a multiplicative relationship between two quantities can be expressed as a ratio or a fraction (KS3)
- Identify and work with fractions in ratio problems

F

Solve problems with ratio

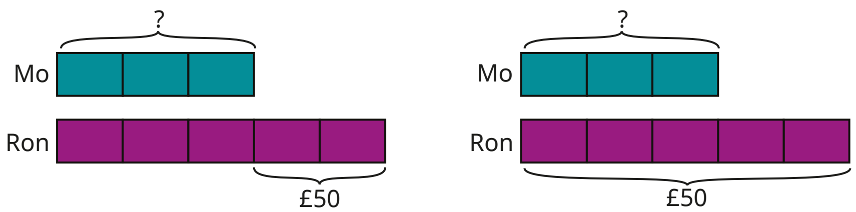
Teaching approaches

- Present two similar ratio problems.

A	B
Mo and Ron share money in the ratio 3 : 5 Ron receives £50 How much does Mo receive?	Mo and Ron share money in the ratio 3 : 5 Ron receives £50 more than Mo. How much does Mo receive?

Ask students to discuss what is the same and what is different.

Show students two bar models and encourage them to discuss which problem is represented by each one.



Ask questions to deepen students' understanding of each problem.

- What is the value of one part in problem A/B?
- How could we work out the total amount of money shared?
- How can we check if the answers are correct?

Key vocabulary

ratio	comparison of two or more values
share	divide a quantity into equal-sized groups or parts
equal parts	parts of a whole with the same value
whole	total or complete amount

Links and next steps

- Support curriculum – Year 9 Spring Block 4 – Step 3 – Ratio problems (whole, part or difference given)
- Main curriculum – Year 10 Autumn Block 5 – Step 2 – Share in a ratio (total given)
- Main curriculum – Year 10 Autumn Block 5 – Step 3 – Share in a ratio (part or difference given)
- Challenge students to solve problems where a ratio changes. For example, Mo and Ron share money in the ratio 5 : 3. Mo gives Ron £20. Mo and Ron now have the same amount. How much money did Mo have originally?

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Direct proportion

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Notes and guidance

In this small step, students will solve problems involving direct proportion, preparing them for subsequent steps in this block. Highlight that a direct proportion relationship is a multiplicative relationship between two values.

Double number lines and ratio tables are useful to highlight direct proportion relationships. Begin by solving problems with unit cost, for example, working out the cost of 8 pens when each pen costs £1.50, and discuss related inverse operations, such as dividing the total cost of some pens by 1.5 to work out the number of pens.

Students should also use scaling strategies to solve problems. For example, if 4 rulers cost £5 then the cost of 8 rulers can be found by multiplying the cost of 4 rulers by 2. Students will work algebraically with direct proportion later in this block.

Misconceptions and common errors

- Students may try to use an additive relationship rather than a multiplicative relationship. For example, if 3 bottles cost £6 then 5 bottles cost £8, adding 2 to the number of bottles and the cost.
- Students may use an incorrect operation, such as multiplying by a value instead of dividing.

Mathematical talk

- What does "scaling" mean?
- What does "unit cost" mean?
- What does it mean if two quantities are directly proportional?
- "It is necessary to work out the unit cost of an item before scaling to the required value." Give an example where this is true. Give an example of where this is not true.
- "Doubling a recipe to feed twice as many people is an example of direct proportion." Explain why this is true.
- "A plumber charging £18 per hour for a job plus a callout charge of £50 is an example of direct proportion." Explain why this is not true.
- When might you use a scaling method to solve a direct proportion problem?
- Where might direct proportion be used in everyday life?

National Curriculum links

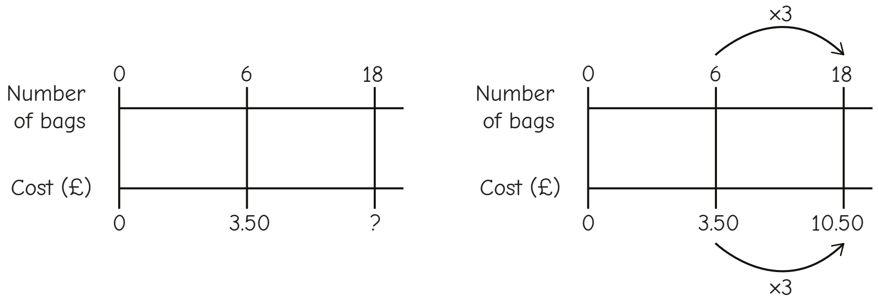
- Solve problems involving direct and inverse proportion, including graphical and algebraic representations (KS3)
- Use scale factors, scale diagrams and maps (KS3)

F Direct proportion

Teaching approaches

- Use a double number line to model how to solve a direct proportion problem.

> 6 bags of crisps cost £3.50
> Work out the cost of 18 bags of crisps.



Tell students that the scale factor can be found by calculating 18 ÷ 6 (= 3) and highlight that this means the number of bags of crisps has been multiplied by 3. Model how to use the double number line to calculate the cost of 18 bags of crisps by multiplying the cost of 6 bags of crisps by 3

Encourage students to discuss why working out the cost of 1 bag may not be an efficient strategy for the problem.

Repeat with other problems using different contexts. For example, distance covered and the time taken for a journey.

Key vocabulary

directly proportional	relationship between two quantities where they increase or decrease at the same rate
scaling	multiplication of each value by a constant to proportionally increase or decrease the size
unit price	cost of a single unit of a product, found by dividing the total price by the number of units

Links and next steps

- Students will need to identify directly proportional relationships such as current and potential difference in a fixed resistor, as described by Ohm's law.
- Support curriculum – Year 9 Spring Block 4 – Step 1 – Direct proportion
- Main curriculum – Year 9 Spring Block 4 – Step 1 – Direct proportion
- Challenge students to link ratio and direct proportion problems. For example, if 3 T-shirts cost £42 then the ratio 3 : 42 (or 1 : 14) can be used to solve problems.

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Best buy problems

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Notes and guidance

In this small step, students will solve financial problems, building on prior learning from Key Stage 3. Provide students opportunities to explore different strategies to compare items and cost, for example, scaling to compare the same amount of each item or working out the unit cost of each item.

It is important to discuss when different strategies may be more efficient than others. Students should also be able to solve problems with common everyday life special offers, for example, "20% off" or "3 for 2" offers.

The use of a calculator is recommended for this step to place more emphasis on strategies to solve problems rather than mental arithmetic.

Mathematical talk

- How much would one of each item cost?
- How can you work out the cost per gram or kg?
- If you divide the cost by the amount, what does this tell you?
- If you divide the amount by the cost, what does this tell you?
- If you compare the unit cost of two items, how do you know which item is better value for money?
- If you compare the amount per pence/pound of two items, how do you know which item is better value for money?
- What is the same and what is different about a "3 for 2 deal" and "$\frac{1}{3}$ off sale"?
- The unit cost of the ______ is £______

Misconceptions and common errors

- Students may misinterpret a smaller amount as being better value for money. For example, thinking 3 g per £1 is better value for money than 4 g per £1
- Students may struggle to compare amounts of money that cannot be physically represented. For example, £0.67845 and £0.67855

National Curriculum links

- Solve problems involving direct and inverse proportion, including graphical and algebraic representations (KS3)
- Use scale factors, scale diagrams and maps (KS3)

F Best buy problems

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Teaching approaches

- Model how to calculate the unit price of two different packs of bottles.

A	B
3 bottles for £3.60	4 bottles for £4.40
£3.60 ÷ 3 = £1.20	£4.40 ÷ 4 = £1.10

Highlight that £1.20 and £1.10 represents the unit price from each pack and ask students which pack is better value for money.

Show students a different strategy using scaling.

A	B
£3.60 × 4 = £14.40	£4.40 × 3 = £13.20

Ask students to discuss what the new amounts represent highlighting that 12 is the lowest common multiple of 3 and 4 and that this method compares the price of 12 bottles using pack A and 12 bottles using pack B.

Repeat with different items and encourage students to discuss which strategy they might use and why.

Key vocabulary

unit price cost of a single unit of a product, found by dividing the total price by the number of units

scaling multiplication of each value by a constant to proportionally increase or decrease the size

directly proportional relationship between two quantities where they increase or decrease at the same rate

Links and next steps

- Support curriculum – Year 9 Spring Block 2 – Step 6 – Best buy problems
- Main curriculum – Year 9 Spring Block 1 – Step 9 – Budgeting
- Challenge students to work out the question from given calculations. For example, given two different-sized bottles and the calculations 150 ÷ 1.35, 1.35 × 7, 3.20 × 3, 350 ÷ 3.20, work out the possible size and price of each bottle.

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Conversion graphs

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Notes and guidance

In this small step, students draw and interpret conversion graphs to complete approximate conversions. Explain to students that if a conversion graph forms a straight line that passes through the origin, it represents a direct proportion. Encourage students to draw a line segment to the graph when reading a value from the axes.

Students can also use the learning from the previous step to work out values that are not in the range of the data represented on a graph. For example, if a graph shows that 5 cm ≈ 2 inches, then 50 cm ≈ 20 inches.

Unfamiliar contexts, such as converting metric units to imperial units, are useful to encourage students to use a graph for their conversions.

Misconceptions and common errors

- Students may read from the wrong axis when completing a conversion with a graph.
- Students may misinterpret different scales on a set of axes.
- Students may struggle to identify multiplicative relationships to convert values outside of the range of a graph.
- When converting metric units, students might rely on their understanding of the multiplicative relationships, which could lead them to bypass the graph.

Mathematical talk

- Why must direct proportion graphs start at the origin?
- Do all conversion graphs start at the origin? Explain your answer.
- Explain why all direct proportion graphs are straight lines.
- What does the horizontal/vertical axis represent?
- What is the same and what is different about a conversion graph and a direct proportion graph?
- Which axis should we read from first? Explain how you know.
- How can you use the graph to convert values outside of the range of a graph?
- If 5 miles ≈ 8 km, then ______ miles ≈ ______ km

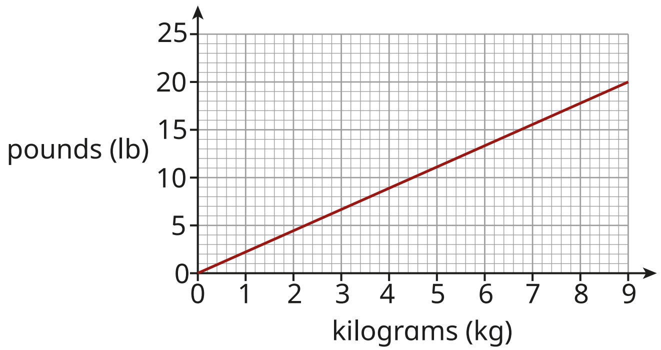
National Curriculum links

- Solve problems involving direct and inverse proportion, including graphical and algebraic representations (KS3)
- Understand that a multiplicative relationship between two quantities can be expressed as a ratio or a fraction (KS3)
- Interpret mathematical relationships both algebraically and graphically (KS3)

Conversion graphs

Teaching approaches

- Show students a conversion graph.



Ask students to use the graph to make approximate conversions.

- How many pounds are there in 10 kilograms?
- How many kilograms are there in 10 pounds?
- Is your answer exact?
- How many kilograms are there in 14 pounds?
- How many kilograms are there in 140/70/35 pounds?

Encourage students to find values by drawing lines accurately from the x-axis to the line and then to the y-axis, and vice versa. Discuss and model appropriate methods for converting values that are not on the graph.

Key vocabulary

directly proportional	relationship between two quantities where they increase or decrease at the same rate
conversion	process of changing a quantity from one unit or form to another
origin	where the axes of a graph intersect, at coordinates (0,0)
linear	relationship or pattern that changes at a constant rate, forming a straight line when graphed

Links and next steps

- Students are expected to recognise a directly proportional relationship on a graph.
- Support curriculum – Year 9 Spring Block 4 – Step 2 – Direct proportion and conversion graphs
- Main curriculum – Year 9 Spring Block 3 – Step 2 – Direct proportion and conversion graphs

F

Exchange rates

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Notes and guidance

In this small step, students will revisit exchange rates, which were covered in Key Stage 3. Explain what an exchange rate implies, for example, £1 = €1.30 means every £1 is worth €1.30

Model strategies to highlight the directly proportional relationship between two values. For example, if the number of euros is found by multiplying the number of pounds by 1.30, then the number of pounds can be found by dividing the number of euros by 1.30

Double number lines can be a useful representation to support students. Ensure that students practise with a variety of currencies. Everyday life contexts are also recommended, for example, comparing prices of items in different countries.

Misconceptions and common errors

- Students may choose the wrong operation when converting between currencies. For example, multiplying by a value rather than dividing.
- Students may not be familiar with foreign currency notation. For example, lira, rupee, Thai baht
- Students may struggle with amounts of money that cannot be physically represented, for example, £42.875
- Students may misinterpret numbers in the context of currency. For example, writing 1.5 instead of £1.50

Mathematical talk

- What does the exchange rate tell us?
- Explain why exchange rates are an example of direct proportion.
- How is the conversion of pounds to euros different to euros to pounds?
- How could we convert the number of pounds into ______? For example, US dollars.
- "Exchange rates are always the same." Is the statement true or false? Explain your answer.
- Will your answer have a greater or smaller numerical value? How do you know?

National Curriculum links

- Solve problems involving direct and inverse proportion, including graphical and algebraic representations (KS3)
- Understand that a multiplicative relationship between two quantities can be expressed as a ratio or a fraction (KS3)
- Use scale factors, scale diagrams and maps (KS3)

F Exchange rates

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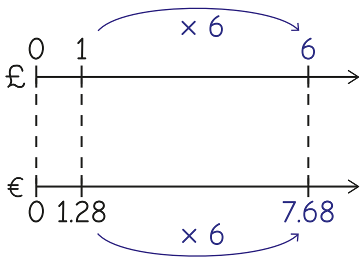
Teaching approaches

- Use a double number line to model how to use an exchange rate.

£1 = €1.28

Convert £6 to euros.

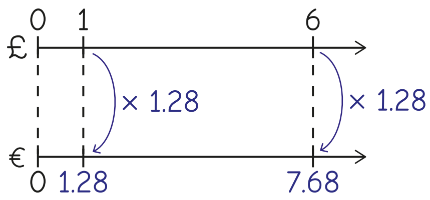
Method 1



1.28 × 6 = 7.68

£6 = €7.68

Method 2



6 × 1.28 = 7.68

£6 = €7.68

Ask students questions to deepen their understanding.

- How could you convert £50 to euros?
- How could you convert €50 to pounds?
- Is €50 greater or less than £50? How do you know?

Repeat with different amounts and exchange rates.

Key vocabulary

exchange rate value at which one currency can be exchanged for another

currency system of money used in a particular country

convert change a quantity from one unit or form to another

Links and next steps

- Main curriculum – Year 9 Spring Block 1 – Step 11 – Spending overseas
- Support curriculum – Year 9 Spring Block 2 – Step 8 – Spending overseas (holidays)
- Main curriculum – Year 8 Autumn Block 2 – Step 5 – Convert between currencies
- Students will use similar strategies to solve problems with direct proportion.
- Challenge students to link ratio and exchange rates. For example, using the ratio 1 : 136 with the exchange rate £1 = ¥136

F

Inverse proportion

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Notes and guidance

In this small step, students explore the relationship between quantities that are inversely proportional. Students should understand that if two quantities are inversely proportional, then when one amount increases, the other amount decreases at the same rate. Students' attention should be drawn to the fact that the product of the two quantities is constant as the quantities vary. It may be useful to begin with a scenario involving one person, such as 1 person takes 4 hours to paint a fence, before moving on to more challenging problems. Encourage students to think about whether a task will take more or less time with more people.

Throughout the step, discuss any assumptions that have been made, such as assuming every person works at the same rate. Foundation students do not need to describe inverse proportion relationships algebraically, for example, $y = \frac{k}{x}$

Misconceptions and common errors

- Students may misinterpret inverse proportion problems and solve them in the same way as direct proportion problems. For example, multiplying both given values by the same value rather than multiplying one and dividing the other.
- Students may assume that any situation in which one quantity decreases represents an inverse relationship.

Mathematical talk

- Will a job always take less time to complete, the more people there are working on it? How do you know? What assumptions have you made?
- As the number of people completing the job ______, the amount of time to complete the job ______ at the same rate. For example, "decreases" and "increases", or vice versa.
- If two variables are inversely proportional, what happens to the value of one variable if the other is multiplied by 5, for example?
- How do you know if a relationship is inversely proportional?
- How can you work out the total amount of time needed for 1 person to complete the job?
- What is the difference between direct and inverse proportion?
- What assumptions have been made? Why is this necessary? How does the assumption affect your answer?

National Curriculum links

- Solve problems involving direct and inverse proportion, including graphical and algebraic representations (KS3)

F

Inverse proportion

White Rose MATHS

Teaching approaches

- Provide students with some information.

  In a competition, £100 is shared equally between the number of winners.

Number of winners	Amount each winner receives
1	100
2	50
4	25
5	20
10	10

Ask students questions about the information in the table.

- Is the number of winners directly proportional to the amount of money each winner receives?
- What happens to the amount of money each winner receives as the number of winners increases?

Encourage students to discuss what they notice about the relationship between the number of winners and the amount of money they each receive, drawing attention to the fact that the number of winners × the amount they each receive is always equal to 100

Key vocabulary

inverse opposite effect of

inversely proportional relationship between two quantities where, as one quantity increases at a rate, the other quantity decreases at the same rate

assumption something accepted as true without proof

Links and next steps

- Main curriculum – Year 9 Spring Block 3 – Step 3 – Inverse proportion
- Support curriculum – Year 9 Spring Block 4 – Step 5 – Inverse proportion
- Challenge students by providing a range of proportion problems to determine whether the problem involves direct proportion or inverse proportion.

F Direct proportion equations E

White Rose MATHS

Notes and guidance

In this small step, students will use their understanding of direct proportion to form equations. Students will be introduced to the symbol $\propto$, which is used to denote when two variables are proportional. For example, $y \propto x$ means y is proportional to x, which implies direct proportion.

Highlight that direct proportion equations are written in the form $y = kx$, where x and y are variables and k is the constant of proportionality. Encourage students to substitute known information into their formula to work out the constant of proportionality. They will then use this constant to work out different corresponding values of the variables.

Students can also solve problems in context. For example, the cost, c, of pens is directly proportional to the number of pens, p.

Misconceptions and common errors

- Students may incorrectly substitute given values to work out the value of k. For example, if $y \propto x$ when $y = 10$ and $x = 5$ then students may write $10 = 5x$ rather than $10 = 5k$
- Students may substitute values incorrectly when forming an equation. For example, writing $24 = 12x$, rather than $12 = 24x$.

Mathematical talk

- How can we write $y \propto x$ as a formula?
- What does k represent in $y = kx$?
- What is meant by the constant of proportionality?
- If a and b are directly proportional, what is the constant of proportionality if $a = 40$ and $b = 25$?
- "$32 = 8k$ is the same as $8 = 32k$"
  Is the statement true or false? Explain your answer.
- How are direct proportion formulae and graphs linked?
- When solving a direct proportion equation, why is it important to first identify the constant of proportionality k?
- What happens to the value of y when x is doubled in a direct proportion equation $y = kx$?

National Curriculum links

- Understand that X is inversely proportional to Y is equivalent to X is proportional to $\frac{1}{Y}$; interpret equations that describe direct and inverse proportion

F Direct proportion equations E

White Rose MATHS

Teaching approaches

- Show students two similar problems to highlight links between direct proportion and algebra.

Mo buys 6 packs of pencils. He has 18 pencils in total. How many pencils are in each pack?	y is directly proportional to x. When $x = 6$, $y = 18$

Using the second problem, model how to use the information to find the constant of proportionality.

$$y \propto x$$
$$y = kx$$
$$18 = 6k$$
$$3 = k$$
$$\longrightarrow \quad y = 3x$$

Explain that $y = 3x$ is a formula linking x and y, and discuss how this relates to the first worded problem.

Ask students to use the formula to work out other values of x and y. For example, work out y if $x = 7$ and x if $y = 144$

Key vocabulary

equation statement to show that two expressions are equal

formula mathematical rule or equation that shows the relationship between different variables or quantities

constant of proportionality value that remains the same when two variables are directly or inversely proportional to each other

Links and next steps

- Students will need to identify directly proportional relationships such as current and potential difference in a fixed resistor, as described by Ohm's law.
- Main curriculum – Year 9 Spring Block 3 – Step 1 – Direct proportion
- Challenge students to form and solve direct proportion equations involving powers. For example, y is directly proportional to x^2

F Direct and inverse proportion graphs E

Notes and guidance

In this small step, students will plot and interpret graphical representations of directly and inversely proportional relationships, which they may have covered in Year 9. Remind students that direct proportion graphs form a straight line from the origin. Highlight that inversely proportional relationships produce a non-linear graph where the horizontal and vertical axes are asymptotes.

When working with direct proportion graphs, ensure students can use proportional reasoning to complete conversions outside of the range of values of the graph. Provide students with plenty of opportunities to identify whether a graph represents a directly proportional or an inversely proportional relationship.

Misconceptions and common errors

- Students may read from the wrong axis when using direct proportion graphs.
- Students may join points with straight-line segments when plotting an inverse proportion graph.
- Students may struggle to recognise the multiplicative relationship between two variables with an inversely proportional relationship.

Mathematical talk

- What are the key features of a direct proportion graph?
- What are the key features of an inverse proportion graph?
- Why do direct proportion graphs always go through the origin?
- Is the graph of an inversely proportional relationship linear or non-linear? Explain your answer.
- Why does an inverse proportion graph have two asymptotes?
- How can you use a graph to work out the constant of proportionality of a directly proportional relationship?
- Describe some everyday life examples of relationships that would produce a direct proportion graph.
- Describe some everyday life examples of relationships that would produce an inverse proportion graph.

National Curriculum links

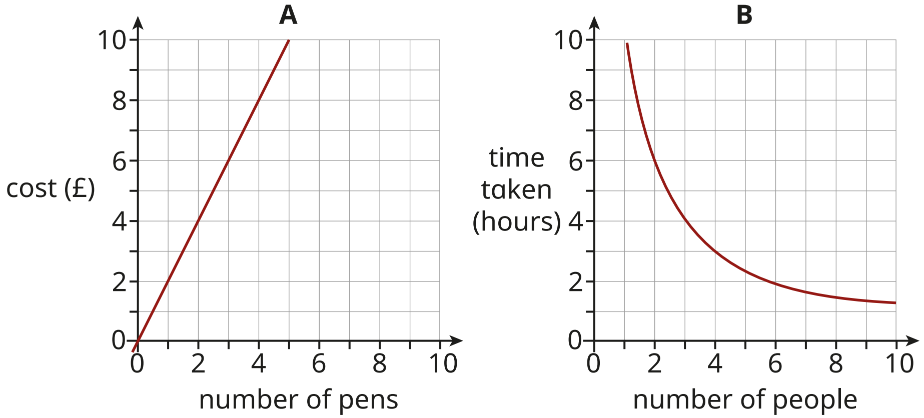
- Solve problems involving direct and inverse proportion, including graphical and algebraic representations (KS3)
- Interpret the gradient of a straight line graph as a rate of change; recognise and interpret graphs that illustrate direct and inverse proportion

F Direct and inverse proportion graphs E

White Rose MATHS

Teaching approaches

- Show students an example of direct proportion and inverse proportion represented graphically.



Ask students questions to deepen their understanding.

- Which graph shows direct proportion? How do you know?
- What are the features of a direct proportion graph?
- What are the features of an inverse proportion graph?
- What is the value of y when x is equal to 4 in each graph?
- What do you notice about the relationship between the x and y values in each graph?
- What is the value of the constant (k) in each graph? How do you know?

Key vocabulary

linear relationship	relationship between variables that is represented by a straight-line graph
non-linear relationship	relationship between variables that is not represented by a straight-line graph
asymptote	line that the graph approaches but never touches
constant	fixed value that does not change

Links and next steps

- Students will identify that two quantities are directly proportional by plotting a graph from experimental data.
- Main curriculum – Year 9 Spring Block 3 – Step 2 – Direct proportion and conversion graphs
- Main curriculum – Year 9 Spring Block 3 – Step 4 – Inverse proportion graphs (E)
- Challenge students to make links between inverse proportion graphs and equations.

Autumn Block 2

Area and volume

White Rose MATHS

Small steps

Step 1	Isometric drawings
Step 2	Plans and elevations
Step 3	Volume of a cylinder
Step 4	Volume of a sphere
Step 5	Volume of cones and pyramids
Step 6	Surface area of a sphere
Step 7	Surface area of a cylinder (E)

Small steps

Step 8	Surface area of a pyramid (E)
Step 9	Surface area of a cone (E)
Step 10	Convert metric units of area (E)
Step 11	Convert metric units of volume (E)

F

Isometric drawings

Notes and guidance

In this small step, students draw and interpret 3-D shapes on isometric paper. Students may be familiar with isometric paper when exploring volumes in cuboids in Key Stage 2. Ensure students know that adjacent dots on isometric paper are equidistant. Start by having students draw cubes and cuboids isometrically made from centimetre cubes, before drawing cubes and cuboids with labelled integer dimensions.

Encourage students to start drawing the shape from its base towards the bottom of the isometric paper, so that they do not run out of room. When ready, have students practise drawing and interpreting triangular prisms isometrically. Having 3-D shapes available will help students to understand how the shape is represented using the isometric paper. Volume of cubes, cuboids and triangular prisms can be interleaved in this step.

Misconceptions and common errors

- Students may draw horizontal lines for shapes that have perpendicular edges.
- Students may count the number of dots instead of the number of gaps between them when drawing specific lengths.
- Students may attempt to draw 3-D shapes in other orientations, for example, ‘upside-down’.

Mathematical talk

- Which part of the shape should you draw first?
- Why can’t horizontal lines be drawn on isometric paper?
- If there are parallel edges on the shape, will those edges be parallel on the isometric drawing?
- If there is a right angle between two edges on the shape, will there be a right angle on the isometric drawing?
- Which 3-D shapes would not be suited to being drawn on isometric paper?
- How does isometric drawing differ from other ways to draw 3-D shapes?

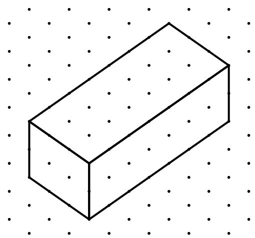
National Curriculum links

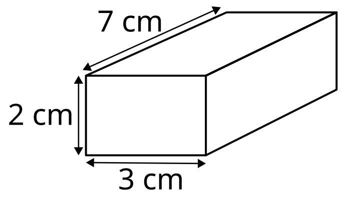
- Use the properties of faces, surfaces, edges and vertices of cubes, cuboids, prisms, cylinders, pyramids, cones and spheres to solve problems in 3-D (KS3)

F Isometric drawings

Teaching approaches

- Show students isometric and perspective drawings of the same cuboid side by side.

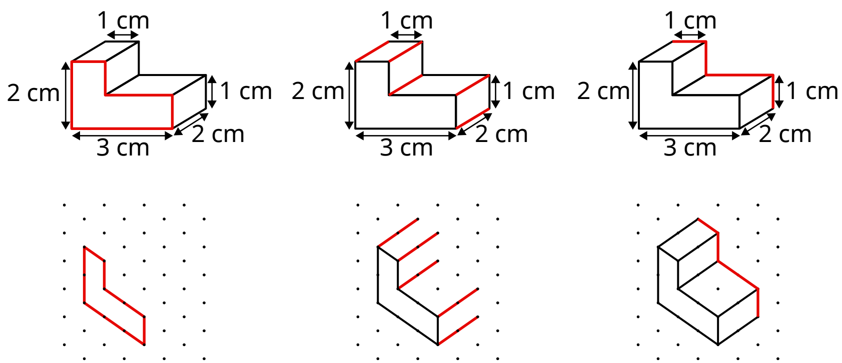




Ask students questions about the drawings.

  - What is the same and what is different about the drawings?
  - What feature does the isometric drawing have that the other does not?

- Model how to draw a 3-D shape on an isometric grid highlighting corresponding edges.



Key vocabulary

isometric drawing way of representing a 3-D object in two dimensions

parallel relationship between two or more objects (like lines or planes) that remain equidistant from each other and never intersect

face flat surface on a 3-D shape

edge where two faces of a shape meet

vertex point at which two or more edges (3-D) or sides (2-D) meet

Links and next steps

- Challenge students to create an isometric drawing of a complex 3-D shape, such as a cuboid with a cut-out section.
- Challenge students to draw the same 3-D shape in different orientations.

F

Plans and elevations

White Rose MATHS

Notes and guidance

In this small step, students will draw accurate and scaled plans as well as front and side elevations of 3-D shapes. They should begin by drawing plans and elevations of cubes and cuboids before moving onto more complex shapes such as prisms. Allowing students access to physical objects such as interlocking cubes to make shapes or dynamic geometry software may be useful to help students visualise objects before attempting to draw plans and elevations.

If appropriate, have students draw or describe shapes when given the plan and elevation views. This includes being able to draw them on isometric paper, which was covered in the previous step.

Misconceptions and common errors

- Students may confuse the different views. For example, drawing a plan rather than a front elevation.
- Students may neglect auxiliary lines that indicate a change in depth.
- Students may use incorrect dimensions to draw the side or front elevations of a triangular prism. For example, drawing a slant height rather than a perpendicular height.

Mathematical talk

- What is the difference between the plan and an elevation?
- Describe what you can see looking at the shape from the front/the side/above.
- Are there any parts of the shape that you cannot see from the front/the side/above?
- How can we show that the faces are at different depths?
- What is the difference between a plan and the net of a 3-D shape?
- If the plan, front elevation and side elevation are the same, what could the shape be?
- How do you know which blocks are hidden or visible in an elevation?
- Do plans and elevations always give you enough information to recreate the 3-D shape exactly? Explain how you know.
- Why might architects or builders use plans or elevations instead of 3-D drawings?

National Curriculum links

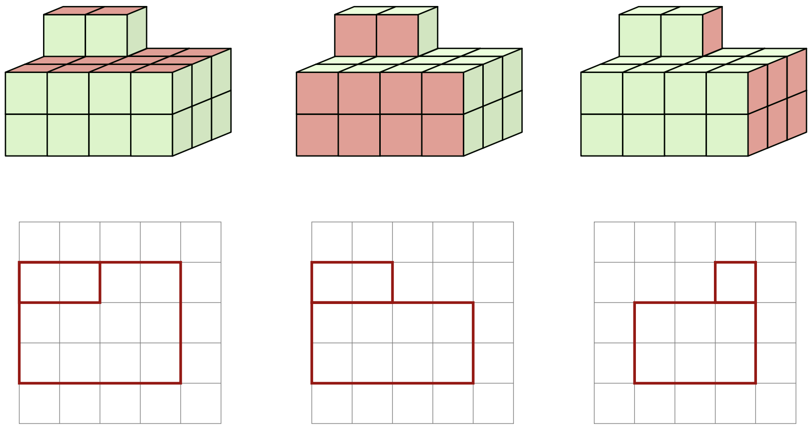
- Construct and interpret plans and elevations of 3D shapes

F

Plans and elevations

Teaching approaches

- Use dynamic geometry software to highlight the faces that correspond to the plan and elevations of a shape.



- Give students five interlocking cubes and ask them to build a shape using the cubes.

  Ask students to draw the plan and elevations for their shape and pass them to a partner.

  Next, ask the partner to construct their shape using the drawings and then check that it is the same as the original.

  Encourage students to explore how different orientations of the same shape affect the plan and/or elevations.

Key vocabulary

plan	2-D diagram that shows the view of a 3-D shape from above
front elevation	2-D diagram that shows the view of a 3-D shape from the front
side elevation	2-D diagram that shows the view of a 3-D shape from the side
dimensions	measurements, such as length, width and height, that describe the size and extent of an object
auxiliary line	line to show a change of depth

Links and next steps

- Understanding of plans and elevations can help students to visualise other problems, such as Pythagoras' theorem, in a 3-D context.
- Challenge students to work out the volume or surface area of a shape given its plan, front elevation and side elevation.

F

Volume of a cylinder

White Rose MATHS

Notes and guidance

In this small step, students will build on prior learning from Year 10, combining their understanding of the area of a circle and the volume of prisms. Highlight that, although cylinders are not prisms, the volume of a cylinder is calculated in a similar way by multiplying the area of the constant cross-section by the length or height.

Allow students the opportunity to practise giving answers exactly in terms of pi (π) and to various degrees of accuracy, for example, rounded to 1 decimal place.

If appropriate, students could solve problems involving the volume of a cylinder, for example calculating an unknown length within the cylinder.

Misconceptions and common errors

- Students may use the diameter rather than the radius when calculating the area of the circular face.
- Students may misinterpret the height of a cylinder when shown in non-standard orientations.
- Students may incorrectly calculate the area of the circular face. For example, multiplying the radius by pi (π) prior to squaring, or squaring pi (π) instead of the radius.
- Students may omit or state incorrect units with their answer.

Mathematical talk

- What is the shape of the constant cross-section of a cylinder?
- How is finding the volume of a cylinder similar to finding the volume of a prism?
- Explain how to find the volume of a cylinder if you are told its height and diameter.
- What are the units of volume for the shape?
- What is the most accurate way to give your answer? How will you know when to do this or not?
- Given the volume and the radius of a cylinder, how would you work out the height of the cylinder?
- Is it possible for a shorter, wider cylinder to have more volume than a taller, thinner one? Explain how you know.
- What everyday objects are shaped like cylinders?

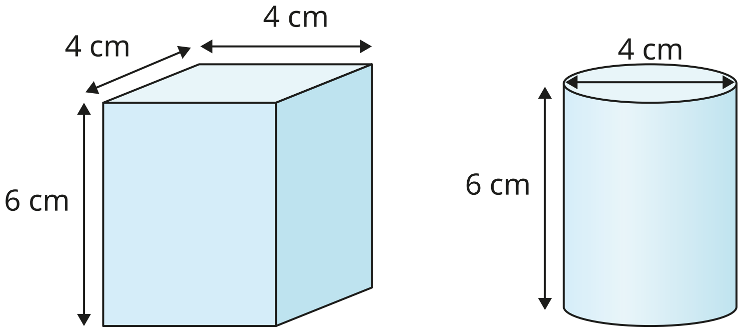
National Curriculum links

- Derive and apply formulae to calculate and solve problems, involving: perimeter and area of triangles, parallelograms, trapezia, volume of cuboids (including cubes) and other prisms (including cylinders) (KS3)
- Use the properties of faces, surfaces, edges and vertices of cubes, cuboids, prisms, cylinders, pyramids, cones and spheres to solve problems in 3-D (KS3)

F Volume of a cylinder

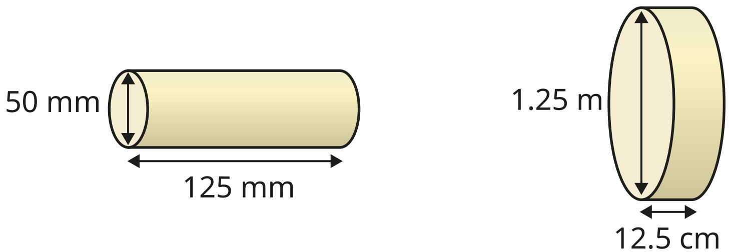
Teaching approaches

- Display a cylinder and a cuboid with the same height alongside each other, such that the diameter of the cylinder is equal to the side lengths of the cuboid's base.



Ask students to consider how they could work out the volume of the cuboid and then encourage students to apply this thinking to calculating the volume of the cylinder.

- Explore finding volumes of cylinders in different orientations, as well as using different units of measurement.



Key vocabulary

volume	amount of space a 3-D object takes up, measured in cubic units
constant cross-section	2-D shape seen when a 3-D object is cut parallel anywhere between a pair of congruent faces
radius	distance from the centre of a circle to any point of the circumference
diameter	straight line passing through the centre of the circle from one point on the circumference to another

Links and next steps

- Main Curriculum – Year 9 Autumn Block 3 – Step 8 – Volume of a cylinder
- Support Curriculum – Year 9 Autumn Block 3 – Step 9 – Volume of a cylinder
- Challenge students to express the volume of a cylinder algebraically, given dimensions in terms of an unknown.
- Students will calculate the volume of various shapes, including cylinders, when they calculate density.

F

Volume of a sphere

White Rose MATHS

Notes and guidance

In this small step, students use the formula to calculate the volume of a sphere. Tell students that the formula to work out the volume of a sphere with radius r is $\frac{4}{3}\pi r^3$, and model how to input relevant calculations into a calculator, for example $\frac{4}{3} \times \pi \times 5^3$, to work out the volume of a sphere with a radius of 5 cm.

As with other formulae involving pi (π), students should have the opportunity to provide exact answers in terms of pi (π), as well as giving approximate answers to varying degrees of accuracy. When students are comfortable with finding the volume of a sphere, they could begin to investigate the volume of hemispheres and compound shapes.

Misconceptions and common errors

- Students may input calculations incorrectly when using a calculator, for example calculating $\frac{4}{3} \times \pi \times 6^2$ rather than $\frac{4}{3} \times \pi \times 6^3$
- Students may incorrectly use the diameter rather than the radius in the formula $V = \frac{4}{3}\pi r^3$
- Students may omit or state incorrect units with their answer.

Mathematical talk

- What do we need to know about the sphere in order to work out the volume?
- How does the volume of a hemisphere compare to the volume of a sphere?
- How could you work out the radius/diameter of a sphere if you know the volume?
- "The radius of sphere A is half the radius of sphere B, therefore the volume of sphere A is half the volume of sphere B".
  Explain why this is not true.
- Why do you think the volume of a sphere depends on the radius cubed rather than squared?
- If the radius of a sphere doubles, how does the volume change? Explain how you know.

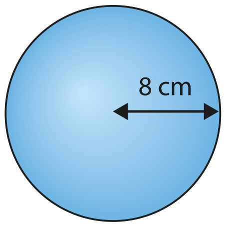
National Curriculum links

- Calculate surface areas and volumes of spheres, pyramids, cones and composite solids

F Volume of a sphere

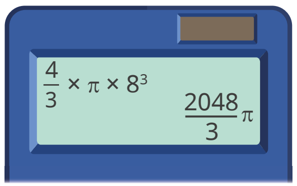
Teaching approaches

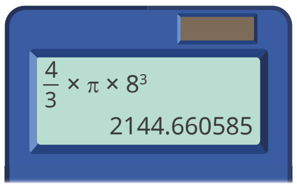
- Show students a sphere and the formula to calculate the volume of a sphere.



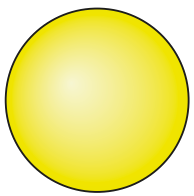
$$V = \frac{4}{3}\pi r^3$$

Use a visualiser to model how to use a calculator to calculate the volume of the sphere.





- Model how to calculate the radius of a sphere when given the volume.



$V = 1500\text{ mm}^3$

$$1500 = \frac{4}{3}\pi r^3$$

$\times 3$

$$4500 = 4\pi r^3$$

$\div 4\pi$

$$\frac{4500}{4\pi} = r^3$$

$\sqrt[3]{\ }$

$$\sqrt[3]{\frac{4500}{4\pi}} = r$$

$r = 7.1$ mm (1 d.p.)

Key vocabulary

volume	amount of space a 3-D object takes up, measured in cubic units
radius	distance from the centre of the sphere to any point on its surface
diameter	distance between two opposite points on the surface of a sphere, passing through the centre
hemisphere	half of a sphere

Links and next steps

- Main Curriculum – Year 9 Autumn Block 3 – Step 9 – Volume of cones, pyramids and spheres E
- Students may later use the volume of a sphere to solve problems involving density.
- Challenge students to write an expression for the volume of a sphere in terms of a variable, for example a sphere with a radius of $3p$ cm.

F

Volume of cones and pyramids

White Rose MATHS

Notes and guidance

In this small step, students calculate the volume of cones and pyramids. Students may not have covered this before, as this is an extend step in Year 9.

When introducing the volume of a cone, it may be useful to draw out similarities with the formula for the volume of a cylinder. That is, the volume of a cone is $\frac{1}{3}$ of the volume of a cylinder with the same height and radius. Then further similarity may be shown between the formulae for cones and pyramids. For example, the volume of both can be calculated with $\frac{1}{3}$ × area of base × height.

If appropriate, students could form and solve equations involving volumes, such as calculating the length of a radius when given the height and volume of a cone.

Misconceptions and common errors

- Students may incorrectly use the slant height of a cone or pyramid rather than the perpendicular height to the base.
- Students may incorrectly use a diameter rather than a radius when calculating the volume of a cone.
- Students may omit or state incorrect units with their answer.

Mathematical talk

- What information do you need to know to be able to calculate the volume of the cone/pyramid?
- How is calculating the volume of a cone similar to calculating the volume of a cylinder?
- What is an apex?
- In the formula for the volume of a cone/pyramid, what is the height measured perpendicular to?
- How is the slant height different to the perpendicular height?
- Why is it important to use the perpendicular height and not the slant height for the volume of cones and pyramids?
- Explain how to calculate the radius of the cone if you know its volume.
- If a cone and a cylinder have the same base and height, why is the cone's volume exactly one-third of the cylinder's?

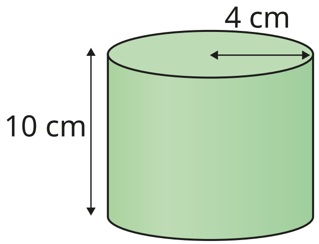
National Curriculum links

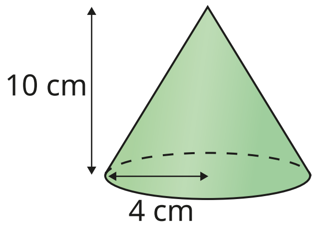
- Calculate surface areas and volumes of spheres, pyramids, cones and composite solids

F Volume of cones and pyramids

Teaching approaches

- Show a cylinder and a cone with the same base, radius and height.

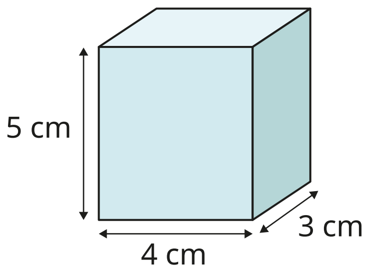


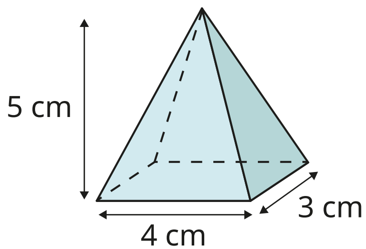


Encourage students to estimate how many times larger the cylinder's volume is compared to the cone's volume.

Show, using dynamic software or otherwise, that the volume of the cylinder is one third of the volume of the cylinder.

Repeat with a cuboid and a pyramid with the same dimensions for height and base.





Key vocabulary

cone 3-D shape with a circular base and a curved surface that narrows to a single point (apex)

pyramid 3-D shape with a polygon base and triangular faces that meet at a point (apex)

perpendicular height measure of how tall a shape is, measured at a right angle from the base to a vertex

slant height distance from one point on the circumference of the base of a cone to the apex

Links and next steps

- Students may be expected to calculate the volume of 3-D shapes in a variety of contexts, such as finding the density of an object.
- Main Curriculum – Year 9 Autumn Block 3 – Step 9 – Volume of cones, pyramids and spheres E
- Students will use volume calculations when using compound measures, such as density.

F

Surface area of a sphere

White Rose MATHS

Notes and guidance

In this small step, students will calculate the surface area of a sphere using the formula. Remind students that the surface area is the area that covers the outside of a 3-D shape, and introduce them to the formula Area = $4\pi r^2$. Although this formula is provided, it may be beneficial to investigate and compare the structure of area and volume formulae using dimensional analysis. For example, area is measured in square units and $4\pi r^2$ involves a length (a radius) being squared.

If appropriate, students could also calculate the surface area of a hemisphere.

Misconceptions and common errors

- Students may apply an incorrect formula, such as confusing surface area with volume.
- Students may make errors when given the diameter of the sphere.
- Students may forget to include the area of a circular base when calculating the surface area of a hemisphere.
- Students may omit or state incorrect units with their answer.

Mathematical talk

- How do you enter the calculation for the surface area of a sphere into your calculator?
- How is finding the surface area of a sphere different from finding its volume?
- If you were given the diameter of a sphere, how would you work out the surface area?
- How does the surface area of a sphere compare to the area of a circle if they both have the same radius?
- If the radius/diameter were doubled/halved, what would happen to the surface area?
- How can the surface area of a sphere be calculated if you are told the volume of the sphere?
- How is the surface area of a sphere similar to and different from the surface area of a cylinder or cone?

National Curriculum links

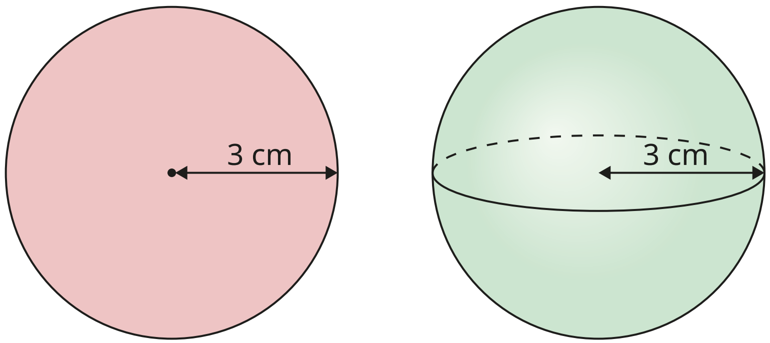
- Calculate surface areas and volumes of spheres, pyramids, cones and composite solids

F Surface area of a sphere

Teaching approaches

- Provide students with the formulae to calculate the area of a circle and the surface area of a sphere.

$A = \pi r^2$ $A = 4\pi r^2$



Ask students questions to deepen their understanding.

- What is the same and what is different?
- What do you notice about the formula for the surface area of a sphere?
- If the area of a circle with a radius of 10 cm is 100π cm^2, what is the surface area of a sphere with the same radius?
- If a sphere has a surface area of 48π cm^2, what is the area of a circle with the same radius?

Key vocabulary

surface area area that covers the outside of a 3-D shape, calculated by finding the total of all its surfaces

hemisphere half of a sphere

radius distance from the centre of the sphere to any point on its surface

diameter distance between two opposite points on the surface of a sphere, passing through the centre

Links and next steps

- Challenge students to work out the surface area of other proportions of a sphere. For example, a quarter of a sphere.
- Explore with students how the formula for the surface area of a sphere is derived.

F Surface area of a cylinder E

White Rose MATHS

Notes and guidance

In this small step, students will learn how to calculate the surface area of a cylinder, which they may have studied previously as an extension step in Year 9.

Exploring the net of a cylinder allows students to see that the length of the curved surface area is equal to the circumference of the circular face. This then supports students to understand the calculations needed to work out the total surface area.

As with previous steps, allow students the opportunity to practise giving answers exactly in terms of pi (π), and to various degrees of accuracy, such as rounded to 1 decimal place.

Misconceptions and common errors

- Students may apply an incorrect formula, such as confusing surface area with volume.
- Students may confuse the radius and the diameter, particularly as they may use both during their calculations.
- Students may only include the area of one circle rather than two.
- Students may omit or state incorrect units with their answer.

Mathematical talk

- What 2-D shapes make up the net of a cylinder?
- If you know the diameter of a cylinder, how would you work out the area of its curved surface?
- How is finding the surface area of a cylinder different from finding its volume?
- When would you not need to include the area of two circles when considering the surface area of a cylindrical shape?
- Can two cylinders with different volumes have the same surface area?
- If you were a manufacturer making cylindrical barrels, would you be more interested in knowing the surface area or the volume? Explain your answer.
- How could surface area of a cylinder be important in everyday life applications like building design or packaging?

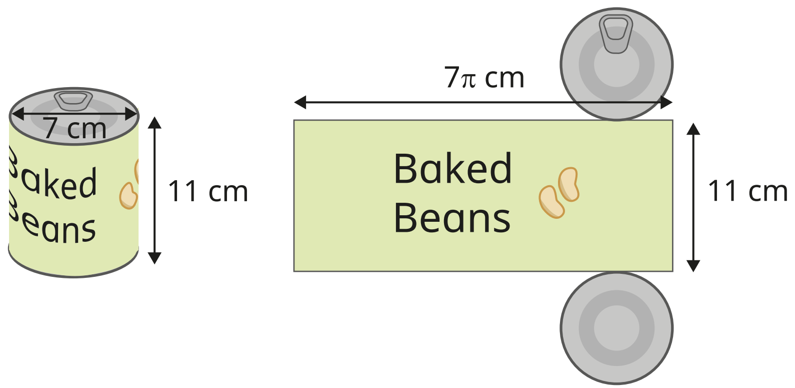
National Curriculum links

- Use the properties of faces, surfaces, edges and vertices of cubes, cuboids, prisms, cylinders, pyramids, cones and spheres to solve problems in 3-D (KS3)

F Surface area of a cylinder E

Teaching approaches

- Display a cylinder and its net to model how to calculate the surface area of the cylinder.



Highlight that the rectangle is the curved surface area of the cylinder and its length is equal to the circumference of one of the circles. Ask students questions to work towards the total surface area of a cylinder.

- How can you work out the area of the rectangle?
- How can you work out the radius of each circle?
- How can you work out the surface area?

Repeat with other cylinders, providing different dimensions such as the radius and height of a cylinder.

Key vocabulary

surface area	area that covers the outside of a 3-D shape, calculated by finding the total area of all its surfaces
circumference	perimeter of a circle
radius	distance from the centre of a circle to any point of the circumference
diameter	straight line passing through the centre of a circle from one point on the circumference to another

Links and next steps

- Main Curriculum – Year 10 Spring Block 5 – Step 6 – Circumference of a circle
- Main Curriculum – Year 10 Spring Block 5 – Step 7 – Area of a circle
- Main Curriculum – Year 9 – Autumn Block 3 – Step 6 – Surface area of a cylinder (E)
- Challenge students to calculate the surface area of compound shapes made from cylinders.

Surface area of a pyramid

F E

White Rose MATHS

Notes and guidance

In this small step, students will learn how to calculate the surface area of a pyramid. Note that students may have calculated the surface area of prisms in Year 10. Students will only need to consider pyramids where the lateral faces are congruent triangles and the apex of the pyramid is directly above the centre of the base.

Nets can be a useful representation to highlight the calculations necessary to work out the surface area of a pyramid. Students need to pay particular attention to given dimensions, for example, ensuring they use the perpendicular height of a triangular face when calculating its area.

Misconceptions and common errors

- Students may use the slant height of a triangular face or the perpendicular height of a pyramid to calculate the area of a triangular face.
- Students may neglect the area of a face when calculating the surface area, for example, not including the area of the base.
- Students may omit or state incorrect units with their answer.

Mathematical talk

- What does the net of a pyramid look like? What 2-D shapes can you identify?
- Which faces are congruent?
- "All pyramids have 5 faces." Give an example of where this is not true.
- How is the slant height different to the perpendicular height?
- What is the least amount of information needed to calculate the surface area of a pyramid?
- How is finding the surface area of a pyramid different from finding its volume?
- What is the same and what is different about the surface area of a square-based pyramid and a triangular-based pyramid?
- How could surface area of a pyramid be important in everyday life applications like building design or packaging?

National Curriculum links

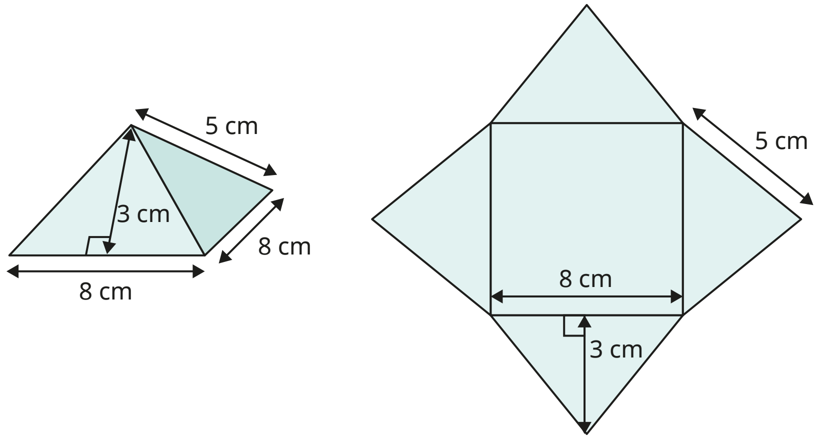
- Calculate surface areas and volumes of spheres, pyramids, cones and composite solids

F Surface area of a pyramid E

White Rose MATHS

Teaching approaches

- Present students with a square-based pyramid.



Ask students questions about the pyramid.

- How many faces does the pyramid have?
- What would the net of the pyramid look like?
- How could you work out the area of each face?

Show students the net of the pyramid and discuss strategies to calculate the surface area of the pyramid. Ensure students' attention is drawn to the perpendicular height of each triangle and the fact that not all dimensions need to be used.

Key vocabulary

pyramid	3-D shape with a polygon base and triangular faces that meet at a point (apex)
lateral faces	side faces of the pyramid that share a common vertex (apex)
perpendicular height	measure of how tall a shape is, measured at a right angle from the base to a vertex
slant height	non-perpendicular height between the vertex at the base of the pyramid and its apex

Links and next steps

- Main Curriculum – Year 10 – Spring Block 5 – Step 9 – Nets
- Main Curriculum – Year 10 – Spring Block 5 – Step 10 – Surface area of a prism
- Challenge students to use Pythagoras' theorem when given the slant height of one of the triangular faces of the pyramid.

F Surface area of a cone E

White Rose MATHS

Notes and guidance

In this small step, students will use a given formula to work out the area of the curved surface of a cone, as well as finding the total surface area. This will normally be given as $A = \pi rl$ where A is the area of the curved surface, r is the radius of the cone and l is the slant height of the cone.

Ensure students understand that the net of a cone consists of a circular base as well as another curved surface, and therefore the total surface area will be the sum of these areas. If appropriate, students could solve other problems involving cones, such as calculating the length of a radius when given the surface area and slant height of a cone.

Misconceptions and common errors

- Students may apply an incorrect formula, such as confusing surface area with volume.
- Students may confuse the radius and the diameter in calculations involving the surface area of a cone.
- Students may use the perpendicular height rather than the slant height to find the curved surface area.
- Students may omit or state incorrect units with their answer.

Mathematical talk

- What does the net of a cone look like?
  What 2-D shapes can you identify?
- How is the slant height different to perpendicular height?
- Why do we need the slant height to calculate the curved surface area instead of the perpendicular height?
- How does the surface area of a cone compare to the surface area of a cylinder with the same base radius and height?
- What is the least amount of information needed to be able to calculate the total surface area of a cone?
- What does your formula sheet give you in relation to the surface area of a cone? Why is it important to consider this?
- How is the curved surface of a cone related to a sector of a circle? How does that help to calculate the surface area?
- How would calculating the surface area of a cone change if the cone was hollow, such as an ice cream cone with no lid?

National Curriculum links

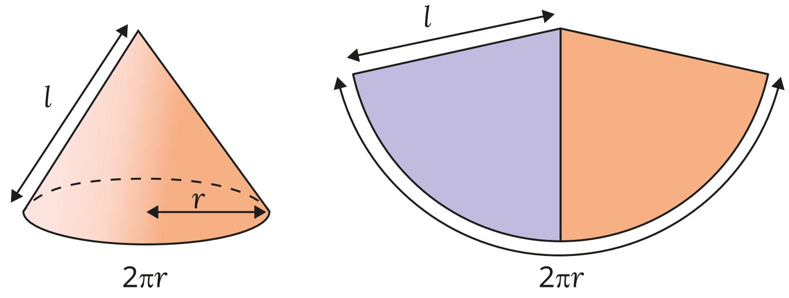
- Calculate surface areas and volumes of spheres, pyramids, cones and composite solids

F Surface area of a cone E

White Rose MATHS

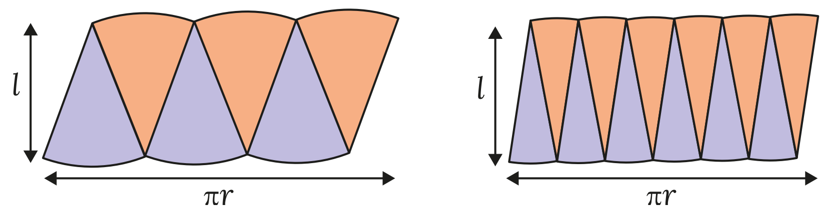
Teaching approaches

- Display a cone and its net labelled with the radius, circumference and slant height.



Shade in the curved surface area equally using two colours, before splitting the curved surface into multiple sectors and arranging them as below.

Highlight to students that if curved surface area is split into smaller equal sectors, then the area approaches $\pi r \times l$.



Key vocabulary

curved surface area	area of the non-flat part of a 3-D shape
cone	3-D shape with a circular base and a curved surface that narrows to a single point (apex)
slant height	distance from one point on the circumference of the base of a cone to the apex
radius	distance from the centre of a circle to any point of the circumference

Links and next steps

- Main Curriculum – Year 10 – Spring Block 5 – Step 9 – Nets
- Main Curriculum – Year 10 – Spring Block 5 – Step 10 – Surface area of a prism
- Challenge students to work with cones with dimensions expressed algebraically.

F Convert metric units of area E

Notes and guidance

In this small step, students convert metric units of area. This may have been previously explored in the extend step in Year 9. Begin by exploring a square with a side length of 1 cm and comparing it to a square with a side length of 10 mm, highlighting that both lengths are equivalent. This leads to the key insight that 1 cm^2 is equal to 100 mm^2

A similar approach can be used to compare other metric units, such as converting between m^2 and cm^2. Encourage students to recognise that the area scale factor is the square of the linear scale factor. Once students are confident, they can progress to converting between units in a single calculation for example, converting 3 cm^2 to mm^2

Mathematical talk

- What are the dimensions of the shape in m/cm/mm?
- What is the area of the shape in m^2/cm^2/mm^2?
- How do you convert from cm^2 to mm^2?
- Does the conversion require a multiplication or a division? Explain how you know.
- Is 1 cm^2 equal to 10 mm^2? Explain how you know.
- What happens to all the dimensions if we change them from metres to centimetres?
- What is the same and what is different when converting units of area compared to converting units of length?

Misconceptions and common errors

- Students may assume the conversion rate is the same between lengths and areas, for example, multiplying by 10 to convert from cm^2 to mm^2
- Students may use the correct scale factor but perform the incorrect operation for the conversion. For example, multiplying by 100 instead of dividing by 100 to convert from mm^2 to cm^2

National Curriculum links

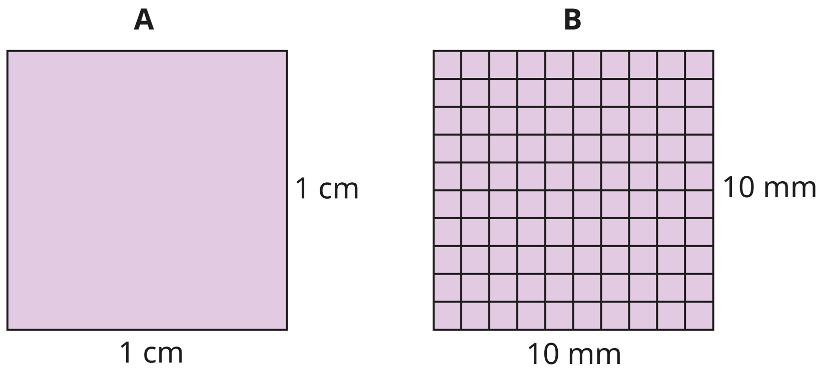
- Change freely between related standard units [for example time, length, area, volume/capacity, mass]
- Use standard units of mass, length, time, money and other measures, including with decimal quantities

F Convert metric units of area E

White Rose MATHS

Teaching approaches

- Display two squares to students.



Ask students questions to deepen their understanding.

- What is the same and what is different about the two squares?
- What is the area of square A?
- What is the area of square B?
- Do the squares have the same area? How do you know?
- How can you convert from cm^2 to mm^2?

Repeat with different units, such as centimetres to metres, encouraging students to consider how they can convert between different metric units of area.

Key vocabulary

dimensions measurements, such as length, width and height, that describe the size and extent of an object

metric system of measurement based on units like metres, grams and litres, used to quantify length, mass and volume

unit standard measurement used to express quantities

area amount of space taken up by a shape or surface, measured in square units

Links and next steps

- Main curriculum – Year 9 Autumn Block 3 – Step 10 – Convert metric units of area and volume (E)
- Students may convert metric units of area when solving problems with pressure, force and area.
- Challenge students to convert units of area given in standard form. For example, convert 4.5×10^7 m^2 to km^2

F Convert metric units of volume E

Notes and guidance

In this small step, students convert metric units of volume. This may have been previously explored as an extend step in Year 9. Similar to the previous step, students start by comparing the volume of a cube with a side length of 1 cm and another cube with a side length of 10 mm. This supports students in understanding that 1 cm^3 is equal to 1000 mm^3

Links between other measurements of volume, such as m^3 and cm^3, should also be explored. Encourage students to recognise that the volume scale factor is the cube of the linear scale factor. Students can then progress to converting between units in a single calculation.

Mathematical talk

- What are the dimensions of the shape in m/cm/mm?
- What is the volume of the shape in m^3/cm^3/mm^3
- How do you convert from cm^3 to cm^3?
- Does the conversion require a multiplication or a division? Explain how you know.
- What happens to all the dimensions if we change them from metres to centimetres?
- Explain why 1 cm^3 is not equal to 10 mm^3
- What is the same and what is different when converting units of volume compared to converting units of length?

Misconceptions and common errors

- Students may assume the conversion rate is the same between lengths and volumes. For example, multiplying by 100 to convert from m^3 to cm^3
- Students may use the correct scale factor but perform the incorrect operation for the conversion. For example, dividing by 1000 instead of multiplying by 1000 to convert from cm^3 to mm^3

National Curriculum links

- Change freely between related standard units [for example time, length, area, volume/capacity, mass]
- Use standard units of mass, length, time, money and other measures, including with decimal quantities

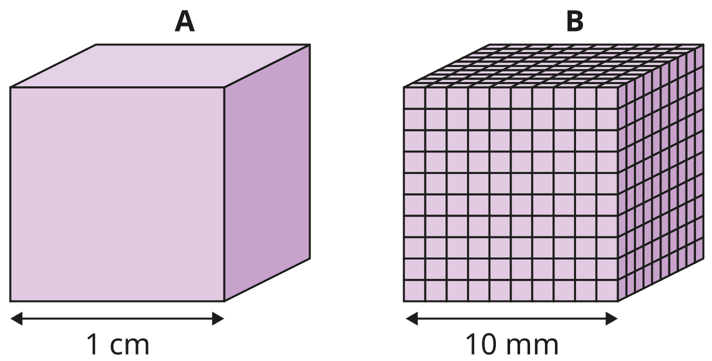
Convert metric units of volume

F E

White Rose MATHS

Teaching approaches

- Display two cubes to students.



Ask students questions to deepen their understanding.

- What is the same and what is different about the two cubes?
- What is the volume of cube A?
- What is the volume of cube B?
- Do the cubes have the same volume? How do you know?
- How can you convert from cm^3 to mm^3?
- Why is the volume of cube B not $10\ mm^3$?

Repeat with different shapes with varying units such as centimetres to metres, encouraging students to consider how they can convert between different metric units of volume.

Key vocabulary

dimensions measurements, such as length, width and height, that describe the size and extent of an object

metric system of measurement based on units like metres, grams and litres, used to quantify length, mass and volume

unit standard measurement used to express quantities

volume amount of space a 3-D object takes up, measured in cubic units

Links and next steps

- Higher tier students will convert between decimetres cubed (dm^3) and centimetres cubed (cm^3) when calculating the concentration of solutions.
- Main curriculum – Year 9 Autumn Block 3 – Step 10 – Convert metric units of area and volume (E)
- Students may later convert metric units of volume when calculating with density.
- Challenge students to solve problems involving the volumes of similar shapes.

Autumn Block 3

Similarity and congruence

Small steps

Step 1	Identify similar shapes
Step 2	Find unknown lengths and angles in similar shapes
Step 3	Identify similar triangles
Step 4	Solve problems with similar shapes
Step 5	Prove a pair of triangles are similar (E)
Step 6	Similarity and congruence
Step 7	Congruent triangles
Step 8	Prove a pair of triangles are congruent (E)

F

Identify similar shapes

White Rose MATHS

Notes and guidance

In this small step, students will revisit their understanding of similar shapes from Key Stage 3. Students will build on their knowledge of enlargement to identify similar shapes. Begin by encouraging students to calculate the scale factor of enlargement for each corresponding length and highlight that if the scale factor is the same, the shapes must be similar. Remind students that angles in corresponding positions in similar shapes are equal. Once students are confident in identifying similar shapes using scale factors, they can begin to incorporate ratios to determine if shapes are similar.

Misconceptions and common errors

- Students may not check that the scale factors are equal for all corresponding sides.
- Students may assume that shapes are similar if they are the same type of shape, without checking corresponding sides have the same scale factor.
- Some students may focus on additive relationships when trying to work out a scale factor. For example, believing that a rectangle is similar to another because both lengths have increased by 3 cm.
- Students may think that shapes in different orientations are not similar.

Mathematical talk

- Explain how you know if two shapes are similar.
- How can you work out the scale factor of enlargement?
- What do you notice about the angles in similar shapes?
- "All rectangles are similar."
  Give an example of when this is true.
  Give an example of when this is not true.
- "Shapes A and B are both rectangles. The shapes must be similar because the lengths of shape B are all 2 cm longer than the lengths of shape A."
  Is the statement true or false? Explain your answer.
- Can shapes be similar if they are in different orientations?
- How can you use ratio to show two shapes are similar/not similar?

National Curriculum links

- Use scale factors, scale diagrams and maps (KS3)
- Apply the concepts of congruence and similarity, including the relationships between lengths, in similar figures

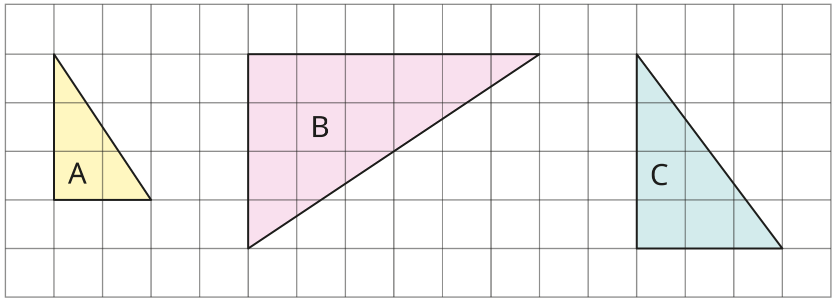
F

Identify similar shapes

White Rose MATHS

Teaching approaches

- Display some right-angled triangles on a grid.



Ask students questions about the triangles.

- Which shapes are mathematically similar?
- What is the scale factor of enlargement from shape A to shape B?
- Why does an additive relationship not represent similarity?
- How could you write the lengths of the perpendicular sides as a ratio for each shape? What do you notice about shape A and shape B?
- What is the scale factor of enlargement from shape B to shape A? Why is this different than from shape A to shape B?

Key vocabulary

scale factor number used to multiply the dimensions of a shape or object to make it larger or smaller compared to its original size

proportion relationship between two or more quantities where the ratio of one quantity to another is the same

similar shapes shapes that have the same angle size in corresponding positions and proportional side lengths, meaning one is an enlargement of the other

Links and next steps

- Support curriculum – Year 9 Spring Block 6 – Step 1 – Recognise similar shapes
- Main curriculum – Year 8 Autumn Block 2 – Step 5 – Similar shapes
- Challenge students to determine if shapes are similar given algebraic lengths.

F

Find unknown lengths and angles in similar shapes

White Rose MATHS

Notes and guidance

In this small step, students calculate unknown angles and lengths in similar shapes, building on learning from Key Stage 3. Ensure students identify corresponding sides correctly, drawing attention to examples where the shapes have different orientations.

Model strategies to calculate the scale factor of enlargement between the shapes, ensuring students can work with both integer and fractional scale factors, for example the scale factor from shape A to B and shape B to A.

Students should be aware that corresponding angles in similar shapes are equal. Encourage the use of geometric notation when labelling side lengths and angles.

Misconceptions and common errors

- Students may assume that two lengths are corresponding if shapes are in different orientations, and incorrectly calculate the scale factor.
- Students may find the difference between corresponding lengths instead of the scale factor.
- Students may think that angles are also affected by a scale factor of enlargement.

Mathematical talk

- Which lengths and angles correspond to each other? How do you know?
- How can you calculate the scale factor of enlargement?
- Why does the order of the letters matter when using geometric notation to refer to angles? For example, writing ABC or BAC.
- Why are corresponding angles in similar shapes equal?
- Is the scale factor from shape A to shape B the same as the scale factor from shape B to shape A? Explain how you know.
- Should the scale factor always be an integer? Explain your answer.

National Curriculum links

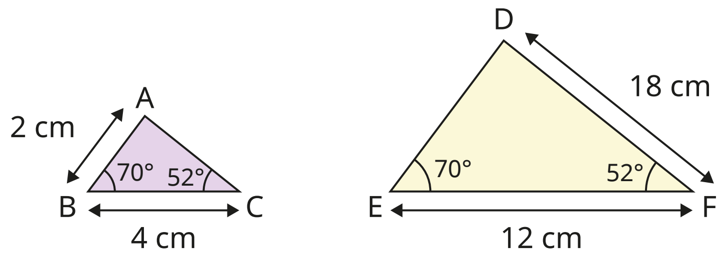
- Use scale factors, scale diagrams and maps (KS3)
- Apply the concepts of congruence and similarity, including the relationships between lengths, in similar figures

F Find unknown lengths and angles in similar shapes

White Rose MATHS

Teaching approaches

- Display two similar shapes to students.



Ask students questions to deepen their understanding.

- How can you use scale factors to calculate length DE?
- How can you calculate the length of AC?
- Why are corresponding angles in similar shapes equal?

Model two different approaches using ratios that compare different lengths to find the length of DE.

AB : BC
2 : 4
= 1 : 2
× 6 (1 : 2) × 6
DE : 12
DE = 6 cm

BC : EF
4 : 12
= 1 : 3
× 2 (1 : 3) × 2
2 : DE
DE = 6 cm

Ask students to consider when each approach may be useful.

Key vocabulary

scale factor	number used to multiply the dimensions of a shape or object to make it larger or smaller compared to its original size
corresponding	elements, such as angles or sides, that are in the same relative position
proportion	relationship between two or more quantities where the ratio of one quantity to another is the same

Links and next steps

- Support curriculum – Year 9 Spring Block 6 – Step 2 – Work out unknown lengths and angles in similar shapes
- Main curriculum – Year 9 Spring Block 5 – Step 2 – Work out unknown lengths and angles in similar shapes
- Students will later calculate unknown lengths to enlarge a shape by a given scale factor.
- Challenge students to calculate unknown lengths when given lengths are expressed algebraically.

F

Identify similar triangles

White Rose MATHS

Notes and guidance

In this small step, students apply their knowledge of angle facts to show that a pair of triangles are similar.

Remind students of facts involving angles that are vertically opposite, angles that form a straight line and angles that exist within pairs of parallel lines. Ensure students see a variety of examples including triangles that share a vertex as well triangles embedded within each other, sharing a common angle.

Support students to identify the corresponding vertices of two similar triangles. Highlight that if all three angles in one triangle are the same as the angles in another, then the triangles are similar. If necessary, it may be appropriate to encourage students to draw the triangles separately and in the same orientation before working out unknown lengths and angles. Students should also recognise that using side ratios is an equally valid method of establishing similarity.

Mathematical talk

- How do you know if two triangles are similar?
- Why do you only need two pairs of equal angles to show that two triangles are similar?
- What is the same and what is different about the pairs of triangles?
- "Two triangles that share a vertex are similar if they include a pair of parallel lines." Give an example of where this is true. Give an example of where this is not true.
- Which diagrams include pairs of parallel lines? How do we show they are parallel?
- Which length corresponds to ______?
- Which angle corresponds to ______?

Misconceptions and common errors

- Students may incorrectly identify corresponding lengths, angles or vertices.
- Students may incorrectly identify alternate and corresponding angles.

National Curriculum links

- Understand and use the relationship between parallel lines and alternate and corresponding angles (KS3)
- Apply the concepts of congruence and similarity, including the relationships between lengths, in similar figures

F

Identify similar triangles

White Rose MATHS

Teaching approaches

- Provide students with two diagrams.

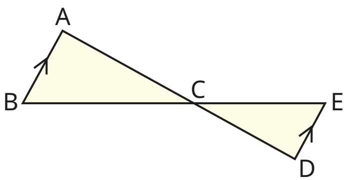


diagram 1

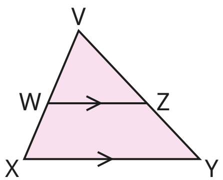


diagram 2

Ask students to label any angles they know are equal in each diagram, giving reasons for their answers.

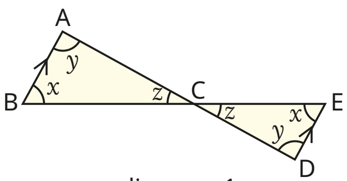


diagram 1

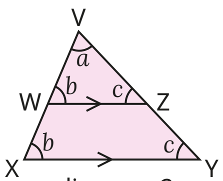


diagram 2

Ask students questions to deepen their understanding.

- Which length in diagram 1 corresponds to AB? How do you know?
- Which length in diagram 2 corresponds to WZ? How do you know?
- Explain why the length VW corresponds to VX.
- Can you work out the scale factor of enlargement from shape CDE to shape ABC? Why or why not?

Key vocabulary

corresponding angles pairs of angles that are in the same relative position when parallel lines are cut by a transversal

alternate angles pairs of angles that are formed by parallel lines cut by a transversal, positioned on opposite sides of the transversal

similar shapes shapes that have the same angle size in corresponding positions and proportional side lengths, meaning one is an enlargement of the other

Links and next steps

- Main curriculum – Year 9 Spring Block 5 – Step 3 – Solve problems with similar triangles (E)
- Students will later use similar triangles to solve bearings problems.
- Challenge students to solve problems with angles given algebraically.

F Solve problems with similar shapes

White Rose MATHS

Notes and guidance

In this small step, students consolidate and extend their understanding of similar shapes.

Begin with a mixture of questions from previous steps where students calculate scale factors between similar shapes, determining if two shapes are similar and calculating unknown lengths and angles. Once students are confident, include problems involving similar triangles.

Demonstrate strategies to help visualise the problem, such as drawing any shapes with a common vertex or angle as two separate shapes in the same orientation. Students should be encouraged to discuss their approaches and reasoning when solving the problems. It may be appropriate to interleave other topic areas such as Pythagoras' theorem or trigonometry.

Misconceptions and common errors

- Students may incorrectly identify the length/angle to calculate when written in geometric notation.
- Students may struggle to identify corresponding sides when shapes are in different orientations.
- Students may struggle to visualise a shape within a shape as two separate shapes.

Mathematical talk

- How do you know if two shapes are similar?
- Does the diagram include pairs of parallel lines? Are any of the angles alternate, corresponding or co-interior?
- How can redrawing the shapes help to visualise the problem?
- Which angles or lengths do you know the size of?
- How do you know if any of the angles are equal? What angle rule did you apply?
- How does a scale factor impact angles in similar shapes?

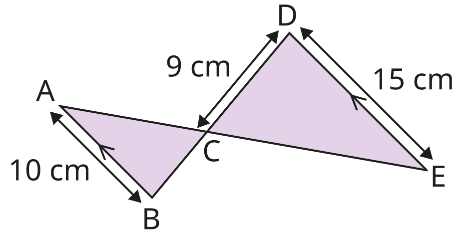
National Curriculum links

- Apply the concepts of congruence and similarity, including the relationships between lengths in similar figures
- Compare lengths, areas and volumes using ratio notation and/or scale factors; make links to similarity (including trigonometric ratios)

Solve problems with similar shapes

Teaching approaches

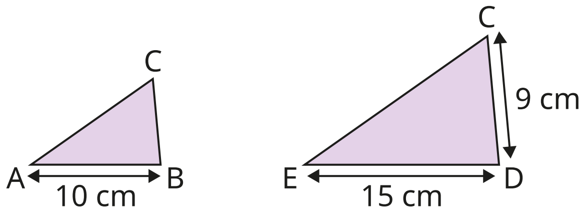
- Display a shape made of two similar triangles.



Prompt students with questions to facilitate discussion.

- What do you know and what can you find out?
- Which angle is the same as angle CDE?
  How do you know?
- What angle rules can be applied here?

Model to students how the shapes can be drawn separately and in the same orientation to facilitate calculating unknown lengths.



Key vocabulary

corresponding angles pairs of angles that are in the same relative position when parallel lines are cut by a transversal

alternate angles pairs of angles that are formed by parallel lines cut by a transversal, positioned on opposite sides of the transversal

similar shapes shapes that have the same angle size in corresponding positions and proportional side lengths, meaning one is an enlargement of the other

Links and next steps

- Main curriculum – Year 9 Spring Block 5 – Step 3 – Solve problems with similar triangles (E)
- Students will later use similar triangles to solve bearings problems.
- Challenge students to solve problems involving three or more similar shapes.

F Prove a pair of triangles are similar E

White Rose MATHS

Notes and guidance

In this extend step, students explore general examples of similar triangles and use formal proofs to justify their similarity. They may need reminders about angle rules, such as those involving parallel lines, to help identify equal angles within a shape. Emphasise the importance of using correct geometric notation when labelling angles and lengths in their proofs. Demonstrate how to structure explanations clearly and precisely to strengthen their justifications.

Reinforce that triangles are similar if all corresponding angles are equal or if their corresponding side lengths are proportional. Where appropriate, challenge students to apply Pythagoras' theorem within similar right-angled triangles.

Misconceptions and common errors

- Students may incorrectly apply an angle rule to identify equal angles, such as identifying an alternate angle as a corresponding angle.
- Students may fail to note in their justification that the shared angle is equal within a nested pair of triangles.
- Students may incorrectly identify equivalent lengths in similar triangles.

Mathematical talk

- How do we know if a pair of triangles are similar?
- Can you use any angle rules to identify equal angles? If so, explain how you know they are equal.
- Which lengths in triangle A correspond to the lengths in triangle B?
- "Triangles are only similar if there are a pair of parallel lines present." Give an example of where this is true. Give an example of where this is not true.
- In a right-angled triangle, how can you calculate an unknown length?
- Angle ______ is equal to angle ______ because…

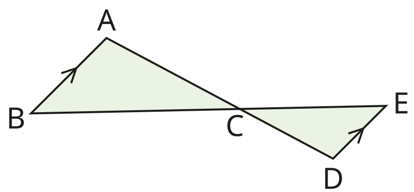
National Curriculum links

- Apply the concepts of congruence and similarity, including the relationships between lengths in similar figures
- Compare lengths, areas and volumes using ratio notation and/or scale factors; make links to similarity (including trigonometric ratios)

F Prove a pair of triangles are similar E

Teaching approaches

- Display a shape made of two similar triangles.



Ask students to match angles that are equal and then choose the correct justification.

∠ABC	∠CDE	vertically opposite angles are equal
∠BAC	∠CED	corresponding angles are equal
∠CAB	∠ECD	alternate angles are equal

Ask students to discuss why this proves that triangles ABC and CED are similar.

Repeat with other pairs of similar triangles, ensuring students use geometric notation to provide clear and concise justifications.

Key vocabulary

corresponding angles pairs of angles that are in the same relative position when parallel lines are cut by a transversal

alternate angles pairs of angles that are formed by parallel lines cut by a transversal, positioned on opposite sides of the transversal

similar shapes shapes that have the same angle size in corresponding positions and proportional side lengths, meaning one is an enlargement of the other

Links and next steps

- Main curriculum – Year 9 Spring Block 5 – Step 3 – Solve problems with similar triangles (E)
- Challenge students to use trigonometry to calculate unknown lengths or angles within similar right-angled triangles to determine similarity.

F

Similarity and congruence

White Rose MATHS

Notes and guidance

In this small step, students explore the differences between similarity and congruence. Remind them that congruent shapes are identical, while similar shapes are proportional to each other. Students should compare different variations of a shape and categorise them to determine if they are congruent, similar or neither.

Ensure students are exposed to shapes in different orientations to encourage them to consider corresponding lengths and angles. By reasoning in this way, students will have a better idea of where the concepts overlap and what characteristics are unique to each.

Mathematical talk

- Explain the differences between similarity and congruence.
- Can two shapes be similar **and** congruent? Explain your answer.
- If you know two shapes are similar, what else do you know about the shapes?
- What is the ratio of corresponding lengths in a congruent shape?
- "If all angles in a shape are equal, then the shapes must be congruent."
  Explain why this is not true.

Misconceptions and common errors

- Students may confuse the definitions of similarity and congruence.
- Students may assume that two shapes are congruent if all angles are equal and no lengths are given.
- Students may assume shapes have to be the same orientation to be congruent.

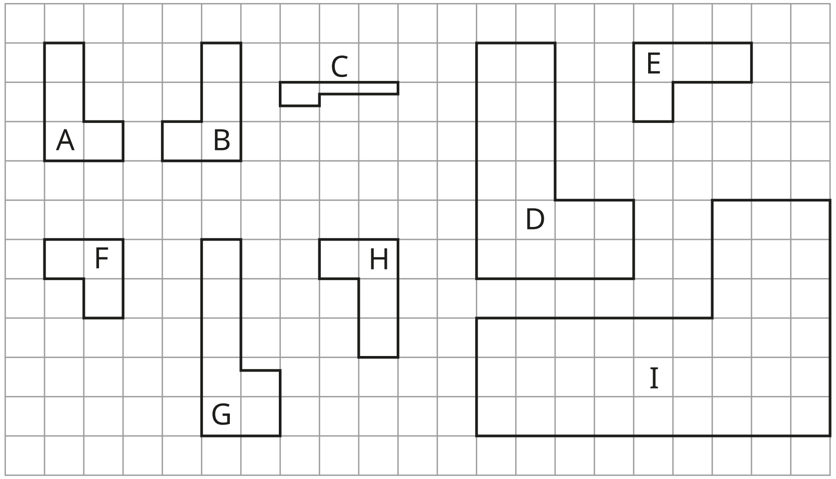
National Curriculum links

- Apply angle facts, triangle congruence, similarity and properties of quadrilaterals to derive results about angles and sides, including Pythagoras' Theorem, and use known results to obtain simple proofs (KS3)
- Apply the concepts of congruence and similarity, including the relationships between lengths in similar figures

Similarity and congruence

Teaching approaches

- Display a grid of shapes to students.



Ask students to sort shapes by whether they are congruent to shape A, similar to shape A, or neither congruent nor similar to shape A. Encourage students to organise this in a table and discuss their reasoning for categorising each shape with a partner.

Congruent to shape A	Similar to shape A	Neither congruent nor similar to shape A

Key vocabulary

congruent identical to, in the context of comparing shapes or solids

similar shapes shapes that have the same angle size in corresponding positions and proportional side lengths, meaning one is an enlargement of the other

scale factor number by which you multiply the dimensions of a shape or object to make it larger or smaller compared to its original size

corresponding elements, such as angles or sides, that are in the same relative position

Links and next steps

- Support curriculum – Year 9 Spring Block 5 – Step 6 – Understand congruence
- Main curriculum – Year 9 Spring Block 4 – Step 8 – Identify congruent shapes
- Challenge students to use understanding of congruence to calculate area and perimeter of compound shapes.

F Congruent triangles

White Rose MATHS

Notes and guidance

In this small step, students will learn how to identify the properties of congruent triangles, which they may have covered in Year 9. It may be necessary to remind students of the conditions of congruence such as SSS, ASA, SAS and RHS.

Begin by defining each condition of congruence, highlighting the key features and showing various examples and non-examples for each. Students should understand the minimum information needed to establish congruence between triangles. Ensure students see examples that are commonly mistaken as congruent but do not present enough information, for example, two triangles with three angles in common are similar, but more information would be needed to determine if the triangles are congruent.

Misconceptions and common errors

- Students may incorrectly identify the condition of congruence.
- Students may not calculate additional information that would allow them to determine a condition of congruence, such as unknown angles.
- Students may incorrectly identify a pair of triangles as congruent by ASA when the given side is not between the two angles.

Mathematical talk

- What conditions of congruence do the triangles satisfy?
- What is the minimum information needed for triangles to be congruent?
- Does it matter which two angles and side are used for the angle-side-angle condition to be true?
- Explain why triangles with three angles in common may not be congruent.
- If the triangles contain a right angle, then the condition of congruence must be "right angle-hypotenuse-side". Give an example of where this is not true.
- Is there any information you need to calculate that will help you decide the condition of congruence?

National Curriculum links

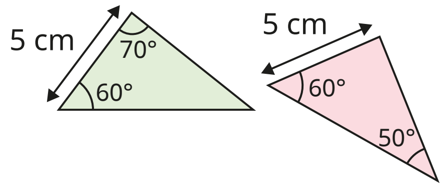
- Use the standard conventions for labelling the sides and angles of triangle ABC, and know and use the criteria for congruence of triangles (KS3)
- Identify and construct congruent triangles, and construct similar shapes by enlargement, with and without coordinate grids (KS3)
- Apply the concepts of congruence and similarity, including the relationships between lengths, in similar figures

F Congruent triangles

Teaching approaches

- Display a pair of triangles to students.

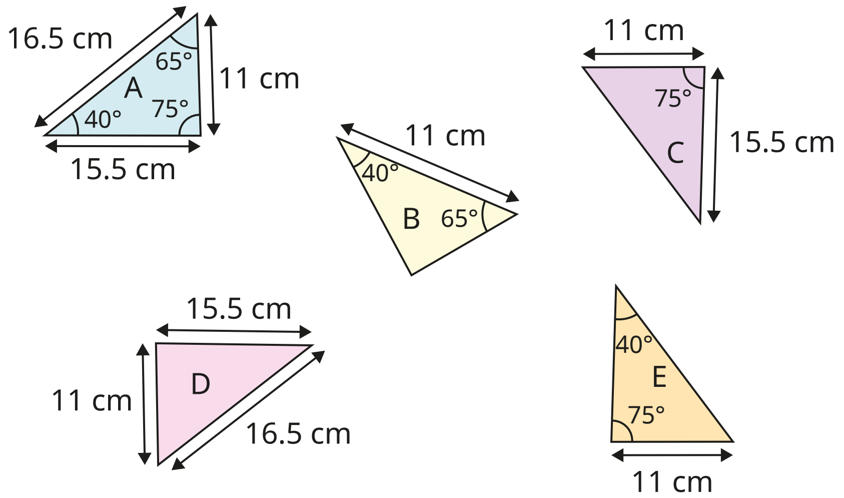
  Ask students questions to encourage their thinking.



  - What do you know and what can you work out?
  - What are the different conditions of congruence?
  - Do any of the conditions of congruence apply to these triangles? Why or why not?

- Display various triangles to students.

  Ask them to justify which shapes are congruent to shape A.



Key vocabulary

congruent identical to, in the context of comparing shapes or solids

condition (congruence) specific set of criteria that must be met for two shapes to be congruent

similar shapes shapes that have the same angle size in corresponding positions and proportional side lengths, meaning one is an enlargement of the other

Links and next steps

- Support curriculum – Year 9 Spring Block 5 – Step 7 – Congruent triangles
- Main curriculum – Year 9 Spring Block 4 – Step 9 – Congruent triangles
- Challenge students to prove that a diagonal in a parallelogram divides it into two congruent triangles.

F

Prove a pair of triangles are congruent

E

White Rose MATHS

Notes and guidance

In this extend step, students perform simple proof using the conditions of congruence and angle facts to prove tringles are congruent. Ensure students are confident in recognising and describing the conditions of congruence. Use examples and non-examples to support students understand and apply full correct statements for angle facts before they start to prove congruence.

Model the process of proof in the first instance and then scaffold by providing partial workings, before students can produce formal proofs independently. Highlight to students that it is important to explicitly write any properties to prove congruence, such as equal lengths or angles, along with correct reasoning. It may be useful to remind students of the properties of special quadrilaterals in preparation for this step.

Misconceptions and common errors

- Students may struggle to make connections to previous topics that would help them identify properties such as equal lengths or angles.
- Students may just write the condition of congruence, for example ASA, without additional reasoning to prove the condition.

Mathematical talk

- Are any lengths equal? How do you know?
- Are any angles equal? How do you know?
- What do you know about angles in parallel lines?
- What does ASA/SSS/SAS/RHS stand for?
- Do any of the conditions of congruence apply to these shapes? How do you know?
- Can you use information provided in the question within your proof?
- Can you prove it any other way using the conditions of congruence?

National Curriculum links

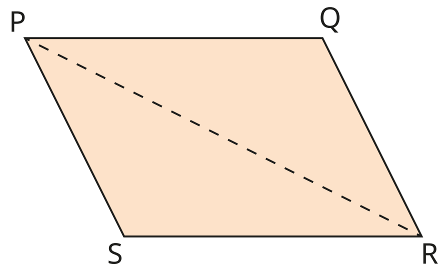
- Apply angle facts, triangle congruence, similarity and properties of quadrilaterals to derive results about angles and sides, including Pythagoras' Theorem, and use known results to obtain simple proofs (KS3)
- Use the standard conventions for labelling the sides and angles of triangle ABC, and know and use the criteria for congruence of triangles (KS3)
- Apply the concepts of congruence and similarity, including the relationships between lengths, in similar figures

F Prove a pair of triangles are congruent E

White Rose MATHS

Teaching approaches

- Display a parallelogram PQRS to students.



Ask students what they know and what they can work out. It may be necessary to encourage discussions around any equal angles or equal lengths.

Provide some partial workings for students to structure their answers.

Length PS is equal to ______ because...

Length SR is equal to ______ because...

Angle PSR is equal to ______ because....

Therefore, triangle PSR is congruent to triangle PQR because ...

Repeat with similar questions, removing the writing frame, and students should become confident structuring their answers.

Key vocabulary

proof	mathematical reasoning presented in logical steps to establish the truth of a statement beyond doubt
congruent	identical to, in the context of comparing shapes or solids
condition (congruence)	specific set of criteria that must be met for two shapes to be congruent

Links and next steps

- Students may later use congruent shapes to solve multi-step problems, including bearings and trigonometry.
- Challenge students to solve problems using angles and algebra by first identifying congruent shapes.